True/False. Write “True” or “False” in the blanks to the left for each statement.

1. **True** A matrix is a rectangular array of numbers.

2. **True** The size of a matrix is the number of rows by the number of columns in that order.

3. **False** A non-square matrix can have an inverse.

4. **True** Matrices allow us to solve systems of equations easier.

5. **False** Matrix multiplication is commutative.

6. **True** We can use \([A]^{-1} [B]\) to determine the solution of a system of equations.

7. **False** A system of equations always has a solution.

8. **True** The matrix which has properties similar to the number “1” is called the identity matrix.

9. **False** Every square matrix has an inverse.

10. **False** Any pair of matrices can be multiplied.

#11. Consider the 2 by 2 system:

\[
\begin{align*}
2x - y &= 5 \\
5x + 2y &= 8
\end{align*}
\]

A. Solve the system by the **substitution method**.

\[
\begin{align*}
2x - y &= 5 \quad \rightarrow \quad y = 2x - 5 \\
5x + 2y &= 8 \quad \Leftrightarrow \quad 5x + 2(2x - 5) &= 8 \\
5x + 4x - 10 &= 8 \\
9x &= 18 \\
\therefore x &= 2, \quad y = -1
\end{align*}
\]
B. Solve the system by the **addition/elimination method**.

\[ 2x - y = 5 \rightarrow 4x - 2y = 10 \Rightarrow 9x = 18 \therefore x = 2 \]

\[ 5x + 2y = 8 \rightarrow 5x + 2y = 8 \]

*also*,

\[ y = -1 \]

\[ \therefore x = 2, y = -1 \]

C. Solve the system by **matrix algebra** \( [A]^{-1} [B] \). Show your coefficient matrix, variable matrix, and constant matrix.

\[ 2x - y = 5 \]

\[ 5x + 2y = 8 \]

\[ A = \begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \]

\[ \begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix} \]

\[ \therefore x = 2, y = -1 \]

D. Graph the two lines and label the intersection point. It should reflect your previous solution point.
#12. Solve the following system using algebraic row reductions and back-substitution.

\[
\begin{align*}
  x + y - z &= 1 \\
  -x + 2y - 3z + 4 &= 0 \\
  3x - 7z - 2y &= 0
\end{align*}
\]

Done in class!

#13. Repeat problem 12 using matrix algebra \([A]^{-1}[B]\). Show your coefficient matrix, variable matrix, and constant matrix.

\[
\begin{align*}
  x + y - z &= 1 \\
  -x + 2y - 3z + 4 &= 0 \rightarrow -x + 2y - 3z &= -4 \\
  3x - 7z - 2y &= 0 \rightarrow 3x - 2y - 7z &= 0
\end{align*}
\]

\[
A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & -3 \\ 3 & -2 & -7 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}
\]

\[
\begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & -3 \\ 3 & -2 & -7 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}
\]

\[
x = 1.75, y = 0, z = 0.75
\]

#14. I had to buy my kids some clothes two months ago. Here is the damage.

3 blouses, 2 skirts, and 4 pair of jeans cost me $292
4 blouses, 1 skirt, and 3 pairs of jeans cost me $252
Oh yeah, each pair of jeans cost $4 more than a blouse.
What did each item cost?

Solve using matrix algebra \([A]^{-1}[B]\). Show your initial system and then your coefficient matrix, variable matrix, and constant matrix.
#15. A long time ago in Liverpool, England, I decided to hire four people to play at my wedding for me. Their names were John, Paul, Ringo, and George. I would pay the entire band a total of $45 to be split between them. After they finished playing and enjoying their refreshments I determined that John deserved twice as much as Paul. Paul and Ringo should get the same while George deserved half as much as what Paul should get. How much did I pay each of these long hair hippie types? Trivia: What is the name of the fifth Beatle and what is Ringo Stars real name? Solve using matrix algebra \([A]^{-1}[B]\). Show your initial system and then your coefficient matrix, variable matrix, and constant matrix.

\[
\begin{align*}
3b + 2s + 4j &= 292 \\
4b + 1s + 3j &= 252 \\
j &= b + 4 - 1b + 0s + 1j = 4 \\
A &= \begin{bmatrix} 3 & 2 & 4 \\ 4 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix}, \quad B &= \begin{bmatrix} 292 \\ 252 \\ 4 \end{bmatrix} \\
\begin{bmatrix} 3 & 2 & 4 \\ 4 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} b \\ s \\ j \end{bmatrix} &= \begin{bmatrix} 292 \\ 252 \\ 4 \end{bmatrix} \\
\end{align*}
\]

\[
\text{blouse} = $29.14, \text{skirt} = $36, \text{jeans} = $33.14
\]

John = $20, Paul = $10, Ringo = $10, George = $5.
#16. Write the partial fraction decompositions for each of the following:

A. \[\frac{4}{2x^2 - 5x - 3}\]

\[
\frac{4}{2x^2 - 5x - 3} = \frac{4}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3}
\]

\[4 = A(x-3) + B(2x+1)\]

\[4 = Ax - 3A + 2Bx + B\]

\[0x + 4 = (A + 2B)x - 3A + B\]

\[0 = A + 2B\]

\[4 = -3A + B\]

Solving

\[B = 4/7, A = -8/7\]

\[\frac{4}{2x^2 - 5x - 3} = -\frac{8}{7(2x+1)} + \frac{4}{7(x-3)}\]

B. \[\frac{x^2 + x}{(x + 2)(x - 1)^2}\]

\[
\frac{x^2 + x}{(x + 2)(x - 1)^2} = \frac{A}{x + 2} + \frac{B}{x - 1} + \frac{C}{(x - 1)^2}
\]

\[x^2 + x = A(x - 1)^2 + B(x + 2)(x - 1) + C(x + 2)\]

\[x^2 + x = Ax^2 - 2Ax + A + Bx^2 + Bx - 2B + Cx + 2C\]

\[1x^2 = Ax^2 + Bx^2 = (A + B)x^2\]

\[1x = -2Ax + Bx + Cx = (-2A + B + C)x\]

\[0 = A - 2B + 2C\]

\[
\begin{bmatrix}
1 & 1 & 0 \\
-2 & 1 & 1 \\
1 & -2 & 2
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} =
\begin{bmatrix}
1 \\
1 \\
0
\end{bmatrix}
\]

\[\begin{bmatrix}
1 & 1 & 0 \\
-2 & 1 & 1 \\
1 & -2 & 2
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix} =
\begin{bmatrix}
1 \\
1 \\
0
\end{bmatrix}
\]

\[A = 2/9, B = 7/9, C = 2/3\]

\[\frac{x^2 + x}{(x + 2)(x - 1)^2} = \frac{2}{9(x + 2)} + \frac{7}{9(x - 1)} + \frac{2}{3(x - 1)^2}\]
C. \( \frac{2x + 4}{x^3 - 1} \)

\[
\frac{2x + 4}{x^3 - 1} = \frac{2x + 4}{(x - 1)(x^2 + x + 1)} = \frac{A}{x - 1} + \frac{Bx + C}{x^2 + x + 1}
\]

\[
2x + 4 = A(x^2 + x + 1) + (Bx + C)(x - 1)
\]

\[
2x + 4 = Ax^2 + Ax + A + Bx^2 - Bx + Cx - C
\]

\[
0x^2 = Ax^2 + Bx^2 = (A + B)x^2
\]

\[
2x = Ax - Bx + Cx = (A - B + C)x
\]

\[
4 = A - C
\]

\[\text{solving}\]

\[
\begin{bmatrix}
1 & 1 & 0 \\
1 & -1 & 1 \\
1 & 0 & -1
\end{bmatrix}
\begin{bmatrix}
A \\
B \\
C
\end{bmatrix}
= \begin{bmatrix}
0 \\
2 \\
4
\end{bmatrix}
\]

\[
A = 2, B = -2, C = -2
\]

\[
\frac{2x + 4}{x^3 - 1} = \frac{2}{x - 1} + \frac{-2x - 2}{x^2 + x + 1} = \frac{2}{x - 1} - \frac{2x + 2}{x^2 + x + 1}
\]