Mat 150 Ch. 6 Practice Test Key **Name ____________________**

True/False. Write "True" or "False" in the blanks to the left for each statement.

1. ***True*** A matrix is a rectangular array of numbers.
2. ***True*** The size of a matrix is the number of rows by the number of columns in that order.
3. ***False*** A non-square matrix can have an inverse.
4. ***True*** Matrices allow us to solve systems of equations easier.
5. ***False*** Matrix multiplication is commutative.
6. ***True*** We can use $[\boldsymbol{A}]^{-1}[\boldsymbol{B}]$ to determine the solution of a system of equations.
7. ***False*** A system of equations always has a solution.
8. ***True*** The matrix which has properties similar to the number "1" is called the identity matrix.
9. ***False*** Every square matrix has an inverse.
10. ***False*** Any pair of matrices can be multiplied.

#11. Consider the 2 by 2 system:

$$2\boldsymbol{x}-\boldsymbol{y}=5$$
$$5\boldsymbol{x}+2\boldsymbol{y}=8$$

A. Solve the system by the **substitution method**.

$$2\boldsymbol{x}-\boldsymbol{y}=5 \rightarrow \boldsymbol{y}=2\boldsymbol{x}-5$$
$$5\boldsymbol{x}+2\boldsymbol{y}=8 \Leftrightarrow 5\boldsymbol{x}+2(2\boldsymbol{x}-5)=8$$
$$5\boldsymbol{x}+4\boldsymbol{x}-10=8$$
$$9\boldsymbol{x}=18$$
$$\therefore \boldsymbol{x}=2, \boldsymbol{y}=-1$$

B. Solve the system by the **addition/elimination method**.

$$2x - y = 5 \rightarrow 4x - 2y = 10 \Rightarrow 9x = 18 \therefore x = 2$$

$$5x + 2y = 8 \rightarrow 5x + 2y = 8$$

$$\textit{also,}$$

$$y = -1$$

$$\therefore x = 2, y = -1$$

C. Solve the system by **matrix algebra** $[A]^{-1}[B]$. **Show your coefficient matrix, variable matrix, and constant matrix.**

$$2x - y = 5$$

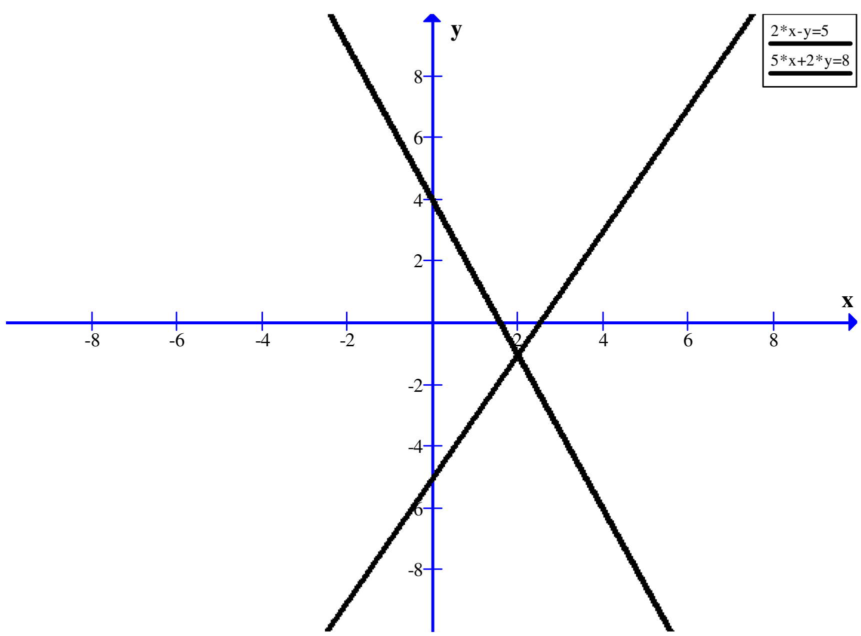
$$5x + 2y = 8$$

$$A = \begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix}, B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 \\ 5 & 2 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

$$\therefore x = 2, y = -1$$

D. Graph the two lines and label the intersection point. It should reflect your previous solution point.



#12. Solve the following system **using algebraic row reductions and back-substitution.**

$$\begin{aligned} x + y - z &= 1 \\ -x + 2y - 3z + 4 &= 0 \\ 3x - 7z - 2y &= 0 \end{aligned}$$

<u>Done in class!</u>

#13. Repeat problem 12 **using matrix algebra $[A]^{-1}[B]$. Show your coefficient matrix, variable matrix, and constant matrix.**

$$x + y - z = 1$$

$$-x + 2y - 3z + 4 = 0 \rightarrow -x + 2y - 3z = -4$$

$$3x - 7z - 2y = 0 \rightarrow 3x - 2y - 7z = 0$$

$$A = \begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & -3 \\ 3 & -2 & -7 \end{bmatrix} B = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 \\ -1 & 2 & -3 \\ 3 & -2 & -7 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 0 \end{bmatrix}$$

$$x = 1.75, y = 0, z = 0.75$$

#14. I had to buy my kids some clothes two months ago. Here is the damage.

3 blouses, 2 skirts, and 4 pair of jeans cost me $292
4 blouses, 1 skirt, and 3 pairs of jeans cost me $252
Oh yeah, each pair of jeans cost $4 more than a blouse.
What did each item cost?

Solve using matrix algebra $[A]^{-1}[B]$. Show your initial system and then your coefficient matrix, variable matrix, and constant matrix.

$$3\boldsymbol{b} + 2\boldsymbol{s} + 4\boldsymbol{j} = 292$$

$$4\boldsymbol{b} + 1\boldsymbol{s} + 3\boldsymbol{j} = 252$$

$$\boldsymbol{j} = \boldsymbol{b} + 4 \rightarrow -1\boldsymbol{b} + 0\boldsymbol{s} + 1\boldsymbol{j} = 4$$

$$\boldsymbol{A} = \begin{bmatrix} 3 & 2 & 4 \\ 4 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} 292 \\ 252 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 2 & 4 \\ 4 & 1 & 3 \\ -1 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{b} \\ \boldsymbol{s} \\ \boldsymbol{j} \end{bmatrix} = \begin{bmatrix} 292 \\ 252 \\ 4 \end{bmatrix}$$

$$\boldsymbol{blouse} = \$29.14, \boldsymbol{skirt} = \$36, \boldsymbol{jeans} = \$33.14$$

#15. A long time ago in Liverpool, England, I decided to hire four people to play at my wedding for me. Their names were John, Paul, Ringo, and George. I would pay the entire band a total of $45 to be split between them. After they finished playing and enjoying their refreshments I determined that John deserved twice as much as Paul. Paul and Ringo should get the same while George deserved half as much as what Paul should get. **How much did I pay each of these long hair hippie types?** Trivia: What is the name of the fifth Beatle and what is Ringo Stars real name? **Solve using matrix algebra $[A]^{-1}[B]$. Show your initial system and then your coefficient matrix, variable matrix, and constant matrix.**

$$\boldsymbol{j} + \boldsymbol{p} + \boldsymbol{r} + \boldsymbol{g} = 45$$

$$\boldsymbol{j} = 2\boldsymbol{p} \rightarrow \boldsymbol{j} - 2\boldsymbol{p} + 0\boldsymbol{r} + 0\boldsymbol{g} = 0$$

$$\boldsymbol{p} = \boldsymbol{r} \rightarrow 0\boldsymbol{j} + 1\boldsymbol{p} - 1\boldsymbol{r} + 0\boldsymbol{g} = 0$$

$$\boldsymbol{g} = \frac{\boldsymbol{p}}{2} \rightarrow 0\boldsymbol{j} - \frac{\boldsymbol{p}}{2} + 0\boldsymbol{r} + 1\boldsymbol{g} = 0$$

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1/2 & 0 & 1 \end{bmatrix} \boldsymbol{B} = \begin{bmatrix} 45 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -2 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1/2 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \boldsymbol{j} \\ \boldsymbol{p} \\ \boldsymbol{r} \\ \boldsymbol{g} \end{bmatrix} = \begin{bmatrix} 45 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\boldsymbol{John} = \$20, \boldsymbol{Paul} = \$10, \boldsymbol{Ringo} = \$10, \boldsymbol{George} = \$5.$$

#16. Write the partial fraction decompositions for each of the following:

A. $\frac{4}{2x^2-5x-3}$

$$\frac{4}{2x^2-5x-3}=\frac{4}{(2x+1)(x-3)}=\frac{A}{2x+1}+\frac{B}{x-3}$$
$$4=A(x-3)+B(2x+1)$$
$$4=Ax-3A+2Bx+B$$
$$0x+4=(A+2B)x-3A+B$$
$$0=A+2B$$
$$4=-3A+B$$
solving
$$B=4/7, A=-8/7$$
$$\frac{4}{2x^2-5x-3}=-\frac{8}{7(2x+1)}+\frac{4}{7(x-3)}$$

B. $\frac{x^2+x}{(x+2)(x-1)^2}$

$$\frac{x^2+x}{(x+2)(x-1)^2}=\frac{A}{x+2}+\frac{B}{x-1}+\frac{C}{(x-1)^2}$$
$$x^2+x=A(x-1)^2+B(x+2)(x-1)+C(x+2)$$
$$x^2+x=A(x^2-2x+1)+B(x^2+x-2)+Cx+2C$$
$$x^2+x=Ax^2-2Ax+A+Bx^2+Bx-2B+Cx+2C$$
$$1x^2=Ax^2+Bx^2=(A+B)x^2$$
$$1x=-2Ax+Bx+Cx=(-2A+B+C)x$$
$$0=A-2B+2C$$
$$\begin{bmatrix}1 & 1 & 0\\ -2 & 1 & 1\\ 1 & -2 & 2\end{bmatrix}\cdot\begin{bmatrix}A\\ B\\ C\end{bmatrix}=\begin{bmatrix}1\\ 1\\ 0\end{bmatrix}$$
$$A=2/9, B=7/9, C=2/3$$
$$\frac{x^2+x}{(x+2)(x-1)^2}=\frac{2}{9(x+2)}+\frac{7}{9(x-1)}+\frac{2}{3(x-1)^2}$$

C. $\frac{2x+4}{x^3-1}$

$$\frac{2x+4}{x^3-1}=\frac{2x+4}{(x-1)(x^2+x+1)}=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}$$

$$2x+4=A(x^2+x+1)+(Bx+C)(x-1)$$

$$2x+4=Ax^2+Ax+A+Bx^2-Bx+Cx-C$$

$$0x^2=Ax^2+Bx^2=(A+B)x^2$$

$$2x=Ax-Bx+Cx=(A-B+C)x$$

$$4=A-C$$

solving

$$\begin{bmatrix}1 & 1 & 0\\ 1 & -1 & 1\\ 1 & 0 & -1\end{bmatrix}\cdot\begin{bmatrix}A\\ B\\ C\end{bmatrix}=\begin{bmatrix}0\\ 2\\ 4\end{bmatrix}$$

$$A=2, B=-2, C=-2$$

$$\frac{2x+4}{x^3-1}=\frac{2}{x-1}+\frac{-2x-2}{x^2+x+1}=\frac{2}{x-1}-\frac{2x+2}{x^2+x+1}$$