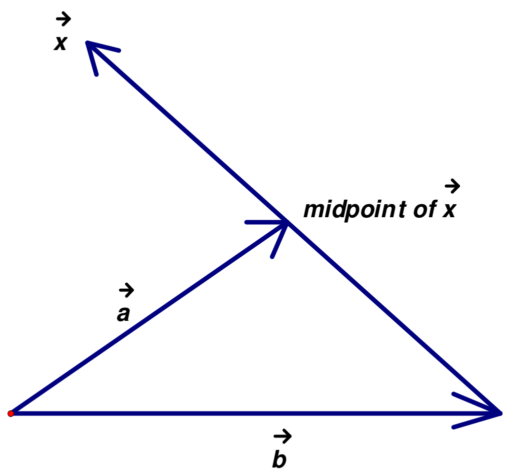
Mat 241 Homework Set 2 – Due ______________

Professor David Schultz

Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are implemented, do so appropriately.**

Mechanics:

#1. Use vector addition to write $\vec{x}$ in terms of $\vec{a}$ and $\vec{b}$. Sketch the appropriate vectors to back up your analysis.



#2. Given the vectors $\vec{a}=\langle 1,2,-2\rangle$, $\vec{b}=\langle 2,-3,1\rangle$ and $\vec{c}=\langle -2,3,6\rangle$, perform the following:

A. **Determine the dot product (or inner product) $\vec{a}\cdot\vec{b}$ and the angle between them.**
B. **Determine both the scalar and vector projections of $\vec{a}$ onto $\vec{c}$.**
C. **Determine the vector projection of $\vec{a}$ perpendicular to the vector direction of $\vec{c}$.**
D. **Determine the direction angles of $\vec{c}$.**
E. **Determine the cross product (or outer product) of $\vec{a}\times\vec{b}$.**
F. **Determine whether $\vec{a},\vec{b}, and\ \vec{c}$ are coplanar.**
G. **Determine if $\vec{c}\times\left(\vec{a}\times\vec{b}\right)=\left(\vec{c}\times\vec{a}\right)\times\vec{b}$.**

#3. Given $\vec{a}=\langle a_1,a_2\rangle$, $\vec{b}=\langle b_1,b_2\rangle$, $\vec{c}=\langle c_1,c_2\rangle$, and the scalar *k*, use the definition of dot product, vector addition, and scalar multiplication to show that in R^2 $\vec{a}\cdot\left(\vec{b}+\vec{c}\right)=\vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ and $\left(k\vec{a}\right)\cdot\vec{b}=k\left(\vec{a}\cdot\vec{b}\right)$.

#4. Given that $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)=7$, use the various properties of dot product and cross product to determine the values of:

- $\vec{b}\cdot(\vec{c}\times\vec{a})$
- $\left(\vec{c}\times(2\vec{a})\right)\cdot\dfrac{\vec{b}}{3}$
- $\vec{b}\cdot\left(b\cdot\times(\vec{a}+\vec{c})\right)$
- $\vec{c}\cdot\left(\vec{b}\times(\vec{a}-2\vec{c})\right)$
-

<u>Concept Development:</u>

#5. State whether each expression is a vector, a scalar, or is meaningless. Note: Each vector is in R^3.

A. $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

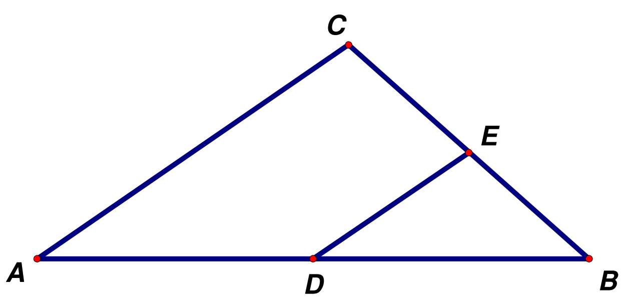
B. $\vec{a}\times\left(\vec{b}\cdot\vec{c}\right)$

C. $\vec{a}\times\left(\vec{b}\times\vec{c}\right)$

D. $\left(\vec{a}\cdot\vec{b}\right)\times\left(\vec{c}\cdot\vec{d}\right)$

E. $\left(\vec{a}\times\vec{b}\right)\times\left(\vec{c}\times\vec{d}\right)$

#6. Use vector addition to show that the segment $\overline{DE}$ which is formed by the midpoints of the two sides of a triangle is parallel to the third side $\left(\overline{AC}\right)$ and half its length. Draw the necessary vectors to illustrate and justify your work.



#7. Prove that $\left(\vec{a}-\vec{b}\right)\times\left(\vec{a}+\vec{b}\right)=2\left(\vec{a}\times\vec{b}\right)$

#8. I had this class in college where the semester's four exams weighed 10%, 15%, 25%, and 50%, respectively. The class average on each of the exams where 75%, 91%, 63%, 87%, respectively. Create two vectors in R^4 to represent the data. Calculate the dot product of your two vectors. What does the scalar value represent in terms of the class?

Bonus Challenge: Let $\vec{a} = \langle a_1, a_2, a_3 \rangle$ and $\vec{b} = \langle b_1, b_2, b_3 \rangle$. prove *Lagrange's Identity,* namely that $\left|\vec{a} \times \vec{b}\right|^2 = \left|\vec{a}\right|^2 \left|\vec{b}\right|^2 - \left(\vec{a} \cdot \vec{b}\right)^2$.

#9. Practice with integration. Compute the following integrals. Sketch the region accurately for part B.

A. $\displaystyle\int_{-\pi}^{\pi} \frac{\sin(z)(z^3 - 2z)}{2z^5 - 3z} dz$

B. Define region *R* as the area in the first quadrant bounded by *y* = cos *x*, *y* = sin *x*, and *x* = 0.

(a) Find the area of *R*.

(b) Calculate the volume generated by revolving *R* about the *y*-axis.

C. Gee, they all look alike.

$\displaystyle\int \frac{1}{x^2 + 6x + 15} dx$ $\qquad$ $\displaystyle\int \frac{x}{x^2 + 6x + 15} dx$ $\qquad$ $\displaystyle\int \frac{x^2}{x^2 + 6x + 15} dx$

Project 1

The project you find below must be typed and turned in along with the above HW. It is not lengthy. Enjoy your new found application of vectors.

We define the angle between two n-dimensional vectors, $\vec{v}$ *and* $\vec{u}$ using the dot product:

$$\cos\theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|\|\vec{v}\|}.$$

We can use this angle to measure how close two populations are to one another genetically. The table shows the relative frequencies of four alleles (variants of a gene) in four populations.

Allele	Eskimo	Bantu	English	Korean
A_1	0.29	0.10	0.20	0.22
A_2	0.00	0.08	0.06	0.00
B	0.03	0.12	0.06	0.20
O	0.67	0.69	0.66	0.57

Let $\vec{a}_1$ be the 4-vector showing the relative frequencies of the alleles in the Eskimo population. Let $\vec{a}_2, \vec{a}_3, \vec{a}_4$ be the corresponding vectors for the Bantu, English, and Korean populations, respectively.

The genetic distance between two populations is defined as the angle between the corresponding vectors.

Question 1: Is the English population closer genetically to the Bantus or to the Koreans. Justify.

Question 2: Is the English population closer to a half Eskimo, half Bantu population than to a Bantu population? Justify.

Question 3: Suppose that we have a population that is x% Eskimo and the remaining percent, (1-x)%, is Bantu. What should the value of x be in order for this mixed population to be as close to the English as possible?
Source: Calculus: Single and Multivariable (Hughes-Hallett, 2005