Mat 241 Homework Set 6 – Due ______________
Professor David Schultz

Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.**

Mechanics:

#1. If $x^y = y^x$ **find** $\frac{dy}{dx}$ **at** $(2,4)$**.**

#2. Given $z^2x^3 + \sin^3\left(yx^2 + yz^3\right) = 2$**, find**

$\frac{\partial z}{\partial x}$

$\frac{\partial z}{\partial y}$

$\left(\frac{\partial z}{\partial x}\right)_y$

$\left(\frac{\partial z}{\partial y}\right)_x$

#3. Suppose $z = f\left(x, y\right), where\ x = r\cos\theta$ **and** $y = r\sin\theta$

Show that:

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = \frac{\partial^2 z}{\partial r^2} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} + \frac{1}{r}\frac{\partial z}{\partial r}$$

#4. Find the equation of the plane tangent to the surface $z = e^{x^2 - y^2}$ **at the point (1, -1, 1). Then use Maple t graph both plane and the surface to verify your result.**

#5. Compute the partial derivatives problems: 15, 21, 22, and 35 found on page 920.

#6. Let $f(x,y)=y\ln x$. Find the gradient of *f*, evaluate the gradient at the point (1, -3), and then find the rate of change in the direction $\vec{u}=\left\langle -\frac{4}{5},\frac{3}{5}\right\rangle$.

#7. Find the directional derivative of $f(x,y,z)=x^2+y^2+z^2$ at $P(2,1,3)$ in the direction of the origin.

Concept Development:

#8. The temperature (in degrees Celsius at a point (x, y, z) in a metal solid is given by:

$$T(x,y,z)=\frac{xyz}{1+x^2+y^2+z^2}$$

A. **Find the rate of change of temperature at (1, 1, 1) in the direction of the origin.**
B. **Find the direction in which the temperature rises most rapidly at (1, 1, 1). Express your answer as a unit vector.**
C. **Find the rate at which the temperature rises moving from (1, 1, 1) in the direction found in part B.**

#9. A certain bug happened to land on a flat plate whose temperature at a point (x, y) is give by $T(x,y)=5+2x^2+y^2$.

A. **Determine the direction the pest should take, starting at (4, 2), in order to cool off as rapidly as possible.**
B. **Plot the temperature function surface *T* using Maple.**

#10. let f_x, f_y, f_{xx}, f_{yy} be continuous and $\vec{u}$ and $\vec{v}$ are constant unit vectors given by $\vec{u}=\langle u_1,u_2\rangle$ & $\vec{v}=\langle v_1,v_2\rangle$. Show that $D_{\vec{u}}\left(D_{\vec{v}}f\right)=D_{\vec{v}}\left(D_{\vec{u}}f\right)$. *Where* $D_{\vec{u}}f=\vec{\nabla}f\cdot\vec{u}$ and $D_{\vec{v}}f=\vec{\nabla}f\cdot\vec{v}$.