

Mat 241 Homework Set 1 – Due _____

Professor David Schultz

Directions: Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.

Mechanics:

#1. Show that the following equation represents a sphere and determine its center and radius.

$$3x^2 - 6x + 3y^2 + 3z^2 + 12z - 6 = 0$$

#2. Find the equation of a sphere if one of its diameters has endpoints given by $(0, -4, 7)$ & $(2, 12, -3)$.

#3. Given the vectors $\vec{a} = \langle a_1, a_2 \rangle$ and $\vec{b} = \langle b_1, b_2 \rangle$ and the scalar, k , use vector addition and scalar multiplication to show that: $k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$ (a kind of distributive law). Make sure you show and justify each step.

#4. Given the points $P_1(1, 1, 7)$ & $P_2(4, 7, 5)$ find the vectors $\overrightarrow{P_1P_2}$ & $\overrightarrow{P_2P_1}$. Compare the two results.

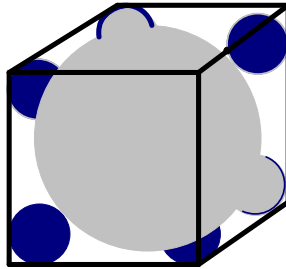
#5. Given the vectors $\vec{a} = \langle 1, 2, -2 \rangle$, $\vec{b} = \langle 2, -3, 1 \rangle$ and $\vec{c} = \langle -2, 3, 6 \rangle$ do the following.

- A. Determine $\vec{d} = 2\vec{a} - \vec{b}$.
- B. Determine the unit vector in the direction of \vec{c} (i.e. find \vec{u}_c).
- C. Determine a vector that points in the opposite direction of \vec{c} and has a magnitude of 10.
- D. Determine a vector that points in the same direction as \vec{c} but is one-half as long.

Concept Development:

#6. If a bug walks on the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 3 = 0$, how close and how far can he get from the origin?

#7. As shown in the accompanying figure, a bowling ball of radius, R , is placed inside a box just large enough to contain it. In order to protect it during shipping 8 little Styrofoam Balls are placed in each corner to absorb shock. What is the radius, r , of the largest Styrofoam ball to be used?



#8. Consider the parabola $y = x^2$. Find the two unit vectors that are parallel to the tangent line at the point $(2, 4)$. I want you to sketch accurately the parabola, tangent line, and two unit vectors on the same graph.

#9. Use vectors to prove that the line joining the midpoints of two sides of a triangle is parallel to the third side and half its length. Provide a sketch with the vectors labeled appropriately.

#10. Find the resultant of the following three forces: 1. a force of 50 N making an angle of 30° with the $+x$ -axis. 2. a force of 80 N making an angle of 135° with the $+x$ -axis, and a force of 30 N making an angle of 240° with the $+x$ -axis.

- A. Draw each vector in standard position on the same coordinate system.
- B. Resolve each vector into its components.
- C. Determine the components of the resultant vector and sketch them on a new coordinate system in standard position.
- D. Draw the resultant vector, \vec{r} , and also determine $|\vec{r}|$ and θ_r .

#11. Sketch on grid paper accurately the following 3-D drawings.

- A. A circle in the xy -plane of radius 2.
- B. The yz -plane.
- C. The surface $x^2 + z^2 = 1$
- D. The surface $y = \sin x$.

#12. Computer Graphics Project 1:

Using a 3-D Graphing Software tool, plot the spheres:

$$x^2 + y^2 + z^2 + 4x - 2y + 4z + 5 = 0 \quad \text{and} \quad x^2 + y^2 + z^2 = 4.$$

A. Determine the equation for the intersection of these two spheres.

B. What shape is it?

C. Add the equation to part A in your graph.

D. Bonus Challenge: What is the volume contained in their intersection?



#13. Practice with integration. Compute the following integrals. Sketch the region for the definite integrals.

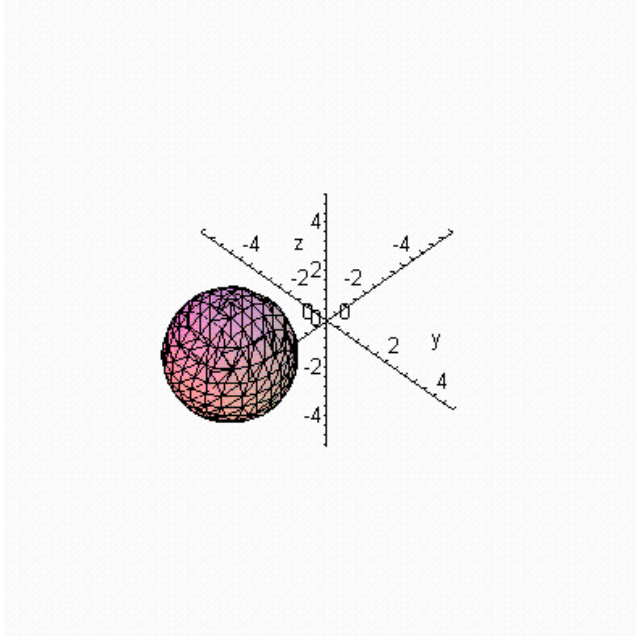
A. $\int_1^e \ln(x) dx$

B. $\int_0^2 \frac{4}{4+x^2} dx$

C. $\int \frac{2x+1}{x^2+2x-3} dx$

D. $\int \frac{dx}{\sqrt{9x^2-16}}$

```
> #Examples using the implicitplot3d command.  
> restart:with(plots):  
Warning, the name changecoords has been redefined  
> implicitplot3d((x-3)^2+(y+1)^2+z^2=4,x=-5..5,y=-5..5,z=-  
5..5,numpoints=10000,axes=normal);
```



```
> implicitplot3d({(x-3)^2+(y+1)^2+z^2=4, 2*x+y-z=2},x=-  
5..5,y=-5..5,z=-5..5,numpoints=10000,axes=normal);
```

