Mat 241 Homework Set 10 – Due _______________

Professor David Schultz

Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.**

Mechanics:

#1. For the following three surfaces identify them by name, sketch them by hand, and then convert them to cylindrical coordinates.

A. $(x-1)^2 + y^2 = 1$

B. $2z^2 = x^2 + y^2$

C. $z = 4 - (x^2 + y^2)$

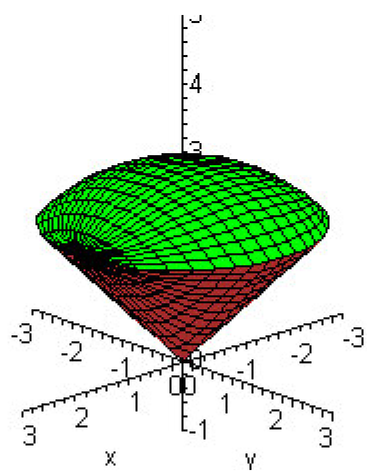
#2. . For the following three surfaces identify them by name, sketch them by hand, and then convert them to spherical coordinates.

A. $x^2 + y^2 + z^2 = 16$

B. $z^2 = \frac{1}{3}(x^2 + y^2)$

C. $z = 2(x^2 + y^2)$

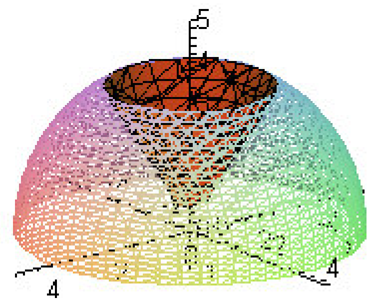
#3. In HW set #7 number 7 we found the volume of an ice-cream cone which was bounded above by the hemisphere $z = \sqrt{8 - x^2 - y^2}$ and below by the cone $z = \sqrt{x^2 + y^2}$. Compute its volume again this time using spherical coordinates.



#4. A solid is bounded above by the upper nappe of the cone $z^2 = 3\left(x^2 + y^2\right)$ and below by the xy – plane and laterally by the sphere given by $x^2 + y^2 + z^2 = 16$. Assume that the density is proportional to the distance from the yz – plane. Use spherical coordinates to find.

A. **The volume of the solid.** $\dfrac{64\pi\sqrt{3}}{3}$.

B. **Mass of the solid.** $\dfrac{32}{3}\left(4\pi + 3\sqrt{3}\right)k$



C. **Check your solution to part A by elementary means (i.e. Mrs. Snaggletooth's 7th grade formulas).**

#5. A solid is formed by cutting the sphere $\rho = 2a$ with the z = 0 and z = a planes. Find the volume of the solid as such:

A. **Using cylindrical coordinates in the order** $dzrdrd\theta$. $V = \dfrac{11\pi}{3}a^3$

B. **Using cylindrical coordinates in the order** $rdrdzd\theta$.

C. **Using spherical coordinates in the order** $\rho^2 \sin\phi d\rho d\phi d\theta$.

D. **By using the washer & shell method from Calculus II.**

#6. A solid is formed from the intersection of the cone $z = \sqrt{x^2 + y^2}$ and the paraboloid $2z = 8 - x^2 - y^2$. Use cylindrical coordinates to find.

A. **The volume of the solid.** $\dfrac{20\pi}{3}$

B. **The z component of the center of mass and the moment of inertia about the z – axis if the density is proportional to the distance to the xz-plane.** $\bar{z} = \dfrac{200}{91}; I_z = \dfrac{2432}{105}k$

<u>Further concept development and applications.</u>

#7. Set up the following volume integrals and sketch the requested region, *R*, in each case. Assume that the density is proportional to the distance from the x = 1 plane.

Solid bounded by: $x+z=2, z=\sqrt{x}, y=3, z=0$

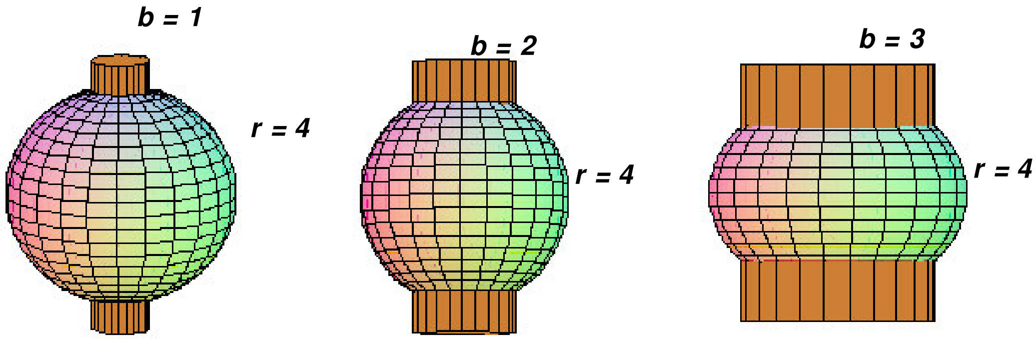
A. **Mass in the dxdzdy order. Sketch R in yz - plane.**
B. M_{yz} **in the dydxdz order. Sketch R in the xz - plane.**
C. I_x **in the dzdxdy order. Sketch R in the xy – plane.**

#8. Determine the surface area of the *entire solid* described in problem 4 (Hint: portion of cone, portion of sphere, and portion of the xy – plane).

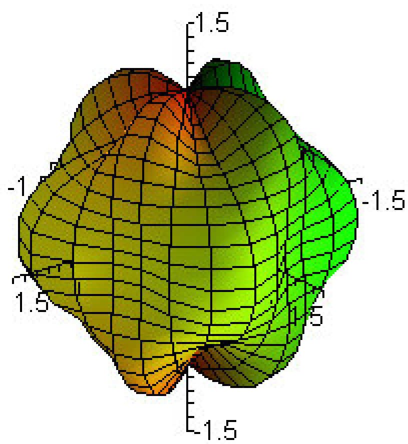
#9.Consider a solid bounded by a paraboloid $z=b\left(x^2+y^2\right)$ **b > 0 and a plane z = h h >0. Use cylindrical coordinates to show that the volume of the solid is** $V=\dfrac{\pi h^2}{2b}$ **and that the centroid of the solid lies on the z – axis and is located at** $\left(0,0,\dfrac{2h}{3}\right)$**.**

These next two problems are not only designed to challenge you, but also to stimulate your brains past what is normally asked. Tackle them with vigor my good people.

Morsel 1.Consider a sphere of radius 4 centered at the origin. Bore a cylindrical hole through the sphere down the z axis with varying radii "b". The solid left over is like a napkin ring. Your task is to determine the volume of the napkin rings for holes bored out of radii 1, 2, and 3. As a final task, how much volume is left over if we assume an arbitrary hole bored out of radius "b" of a sphere of radius "a" (a > b) ?



Morsel 2. Bumpy (or wrinkled) spheres are used to model tumors. The modeling of tumors is of significant importance. Consider the wrinkled sphere given by $\rho = 1 + 0.2\sin(4\theta)\sin(3\phi)$ **where** $0 \le \theta \le 2\pi;\ \ 0 \le \phi \le \pi$**.**



A. Use a computer algebra system (CAS) to approximate the volume.

B. Extra Credit and fun… evaluate the resulting triple integral directly by hand to show that the volume is exactly $\dfrac{3608\pi}{2625} units^3$**.**

Here is the code used to plot the bumpy sphere. Change some values and see what happens!

```
> restart:
> r:=1+0.2*sin(4*x)*sin(3*y):
plot3d([r*sin(y)*cos(x),r*sin(x)*sin(y),r*cos(y)],x=0..2*Pi,y=0..Pi,lig
htmodel=light4,view=[-1.5..1.5,-1.5..1.5,-
1.5..1.5],color=[cos(x),sin(y),tan(z)]);
```

Further reading:"Heat Therapy for Tumors" by Leah Edelstine-Keshet, Summer 1991.