

Mat 241 Homework Set 11 – Due _____

Professor David Schultz

Directions: Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.

Mechanics:

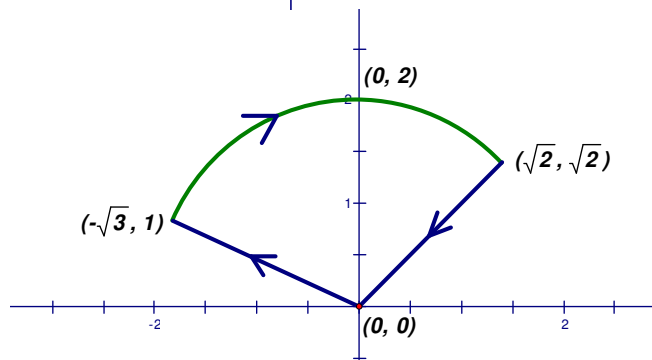
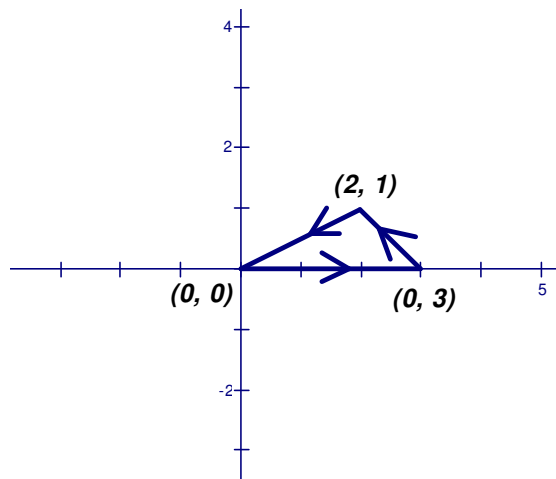
#1. On page 1060 of our text do pbs. 12-18 even.

#2. Find the gradient vector field for $f(x, y, z) = x \cos\left(\frac{y}{z}\right)$.

#3. Given the vector field $\vec{F}(x, y) = \left\langle -\frac{y}{2}, \frac{x}{2} \right\rangle$, do the following:

A. Sketch the field vectors corresponding to the points $(2, 4)$, $(-2, -4)$, $(5, 0)$ and $(0, -5)$.

B. Evaluate the line integral $\oint_C \vec{F} \cdot d\vec{r}$ over the two shown paths.



#3. Using the force $\vec{F} = \langle 2x, -2yz, -y^2 \rangle$ do the following:

A. Determine a scalar function $f(x, y, z)$ such that $\vec{F} = \nabla f$. The function f is called a *potential function*.

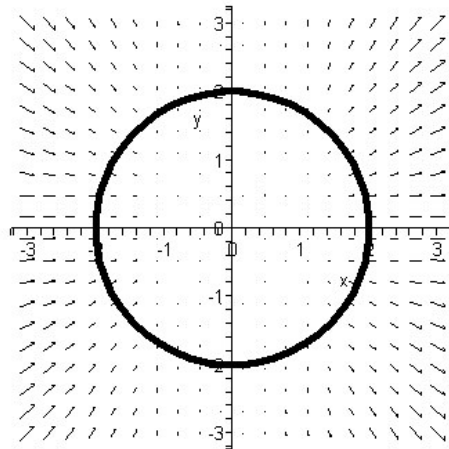
B. Verify that \vec{F} is conservative by showing that $\oint_C \vec{F} \cdot d\vec{r} = 0$ where C is

composed of C_1 which is a portion of the helix

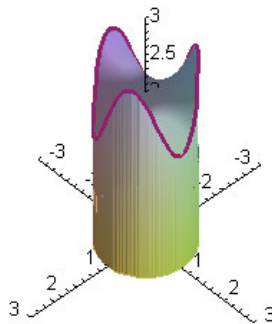
$\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ from $(1, 0, 0)$ to $(0, -1, \frac{3\pi}{2})$ and C_2 is the line

segment from $(0, -1, \frac{3\pi}{2})$ to $(1, 0, 0)$.

#4. Find the work done by the force field $\vec{F}(x, y) = x^2\vec{i} + xy\vec{j}$ on a particle that moves once around a circle centered at the origin of radius 4 oriented in the counterclockwise direction. How does your result play itself out in the picture?



#5. A cylindrical solid piece of bar-stock of radius 1 inch is cut so that the height along the top edge is given by the curve: $z = 2 + 0.5\sin(3t)$ $0 \leq t \leq 2\pi$. (Curve is shown running along the edge of the solid).



Do the following.

1. Determine the lateral surface area by means of a geometric argument.
2. Determine the lateral surface area using a line integral.
3. Suppose you wanted to plate the lateral surface area with nickel plating 1 millimeter thick. According to Prime Plating Co. of Sun Valley, Ca. it costs about 6 cents to plate 1 square inch at that thickness. What would the approximate cost be?
4. Show that the curve $z = 2 + 0.5\sin(3t)$ which traces out the edge of the solid's top lies on the surface $z = 2 + x^2y + 0.5y(x^2 - y^2)$ by using trigonometric identities learned in MAT 182 on $2 + 0.5\sin(3t)$. You should enjoy this!
5. Just for fun...Returning to question 3, suppose you wanted to plate the entire piece's surface. Top, bottom, and lateral side. What would that cost?

#6. This last problem is from my homework set in 1982. If γ is a smooth, but not necessarily simple, closed curve which does not pass through the origin, then the winding number of γ is defined as

$$\frac{1}{2\pi} \int_{\gamma} \vec{F} \cdot d\vec{r} \text{ where } \vec{F}(x, y) = \left(\frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right).$$

It can be shown in advanced calculus texts that the winding number is always an integer which gives the number of times γ winds around the origin. If it is positive, then γ encircles the origin in a counterclockwise sense. If it is negative, then γ encircles the origin in a clockwise sense. Compute the winding numbers for the following two curves.

$$\gamma_1 : \vec{r}(t) = \langle 2\cos t, 2\sin t \rangle \quad 0 \leq t \leq 6\pi$$

and

$$\gamma_2 : \vec{r}(t) = \langle \cos t, -\sin t \rangle \quad 0 \leq t \leq 2\pi$$



Electroless Nickel: Electroless nickel (EN) plating is a process that utilizes an autocatalytic chemical reaction instead of electricity to deposit nickel ions onto a part. This type of plating process gives uniform thickness all over the part's shape and size without the variances that occur in electrolytic plating. This offers a distinct advantage for parts that require very exact tolerances and also offers superior corrosion resistance to electroplated nickel or hard chrome.