Mat 241 Homework Set 12 – Due _____

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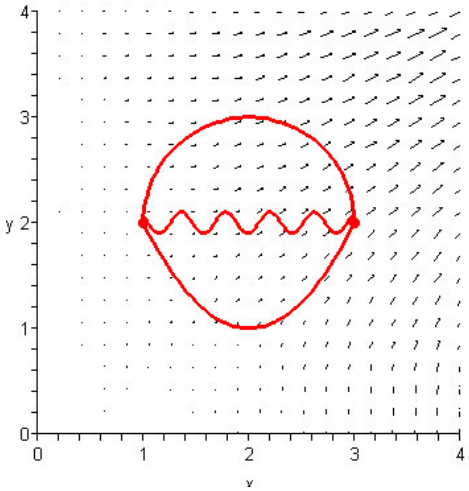
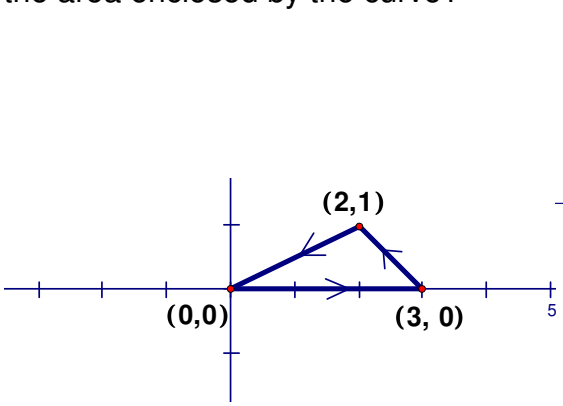
Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.**

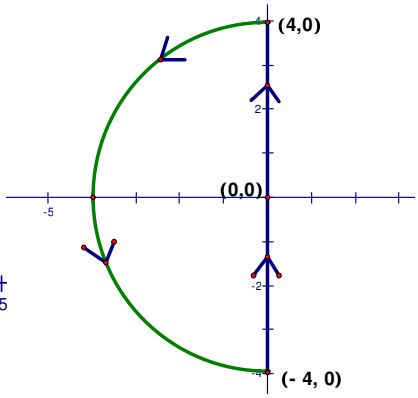
Mechanics:

#1. Consider the vector field given by $\vec{F}(x,y) = 2xy\vec{i} + x^2\vec{j}$ and the three shown paths C_1, C_2, and C_3 where each path starts at (1, 2) and ends at (3, 2). I claim that $\int_C \vec{F} \cdot d\vec{r}$ has the same value for each. Am I telling the truth? Justify your response. What is the line integral's value. What does this imply about $\vec{F}(x,y) = 2xy\vec{i} + x^2\vec{j}$?



#2. Use Green's Theorem to evaluate the following line integrals where $\vec{F} = \left\langle \frac{-y}{2}, \frac{x}{2} \right\rangle$ and C is the given paths shown. How are your results related to the area enclosed by the curve?





#3. Find the work done by the force field $\vec{F}(x,y)=\left(e^{x}-y^{3}\right)\vec{i}+\left(\cos y+x^{3}\right)\vec{j}$ on a particle that travels once around the unit circle in the counterclockwise direction by using Green's Theorem and polar coordinates.

#4. Green's Theorem is significant because it can reduce cumbersome line integrals into a double integral over a region. One must be cautious that all of the conditions are met. This last problem should shed some light.

Why are holes in the Domain of a Vector Field Important?

Let $\vec{F}(x,y)=\dfrac{-y}{x^{2}+y^{2}}\vec{i}+\dfrac{x}{x^{2}+y^{2}}\vec{j}$

A. Calculate $\oint \vec{F}\cdot d\vec{r}$ where C is the unit circle centered at the origin and is traversed once. (Note: the unit circle is traced out by $\vec{r}(t)=\cos t\vec{i}+\sin t\vec{i}$ /*$(x(t)=\cos t, y(t)=\sin t\ \ 0\le t\le 2\pi$.)

B. What is the domain of $\vec{F}$? Sketch the domain by hand.

C. Calculate $\dfrac{\partial Q}{\partial x}-\dfrac{\partial P}{\partial y}$.

D. Calculate our same line integral, $\oint \vec{F}\cdot d\vec{r}$, using Green's Theorem. Sketch the region enclosed by the curve along with the domain found in B on the same set of axes.

D. Compare your answers in parts A & D. Does Green's Theorem even apply?