

Mat 241 Homework Set 16 – Due _____
Professor David Schultz

Directions: Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.

Mechanics:

Use the divergence theorem to evaluate $\iint_s \vec{F} \cdot d\vec{S}$ where the simple solid regions have positive (outward) orientation.

#1. $\vec{F}(x, y, z) = \langle 2x, 2y, 2z \rangle$ with surface $x^2 + y^2 + z^2 = 9$. (ans. 216π)

#2. $\vec{F}(x, y, z) = \langle z^3, -x^3, y^3 \rangle$ with surface $x^2 + y^2 + z^2 = a^2$. (ans. 0)

#3. $\vec{F}(x, y, z) = \langle x - z, y - x, z - y \rangle$ where S is the surface of the cylindrical solid bounded by $x^2 + y^2 = a^2$, $z = 0$ and $z = 1$. (ans. $3\pi a^2$)

#4. $\vec{F}(x, y, z) = \langle x, y, z \rangle$ where S is the surface bounded by the paraboloid $z = 1 - x^2 - y^2$ and the xy - plane. (ans. $3\pi/2$)

#5. $\vec{F}(x, y, z) = \langle x^3 - e^y, y^3 + \sin z, z^3 - xy \rangle$ where S is the surface of the solid bounded by $z = \sqrt{4 - x^2 - y^2}$ and the xy - plane. (ans. $192\pi/5$) – try spherical coordinates.

#6. Let $\vec{F}(x, y, z) = \langle z \tan^{-1}(y^2), z^3 \ln(x^2 + 1), z \rangle$. Find the flux of \vec{F} across the part of the paraboloid $x^2 + y^2 + z = 2$ that lies above the plane $z = 1$ and is oriented upward.