Mat 241 Homework Set 5 – Due ______________

Professor David Schultz

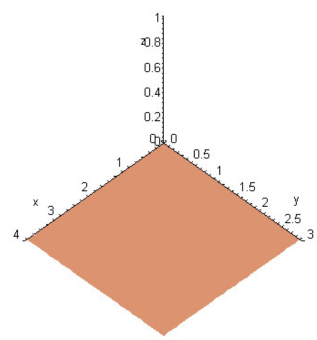
Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are implemented, do so appropriately.**

Mechanics:

#1. Determine & Sketch the domains of the following functions.

A. $f(x,y)=\sqrt{x}+\sqrt{y}$

$x\ge 0 \;\&\; y\ge 0$



The domain is the all non-negative pairs.

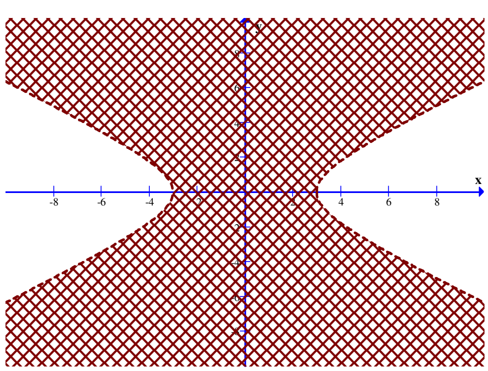
B.

$$g(x,y)=\frac{5}{\left(36-4x^2+9y^2\right)^{\frac{1}{2}}}$$

$$\left(36-4x^2+9y^2\right)^{\frac{1}{2}}>0\Leftrightarrow 36-4x^2+9y^2>0\Leftrightarrow -4x^2+9y^2>-36$$

$$so,\frac{x^2}{9}-\frac{y^2}{4}<1$$

The domain is the interior of the hyperbola.

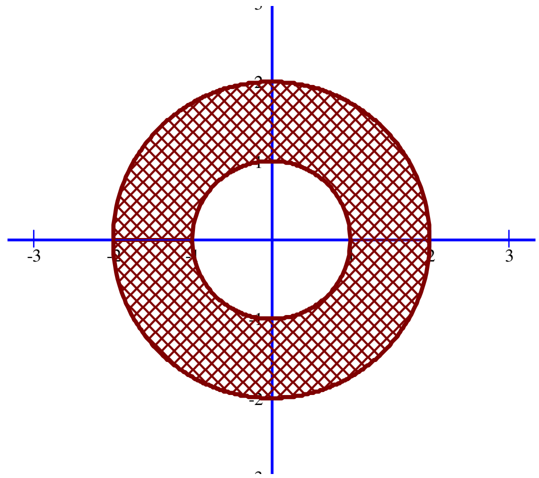


C.

$$f(x,y)=\sqrt{x^2+y^2-1}+\ln\left(4-x^2-y^2\right)$$

$$x^2+y^2-1\geq 0 \text{ and } 4-x^2-y^2>0$$

$$x^2+y^2\geq 1 \text{ and } x^2+y^2\leq 4$$



The domain is the annulus formed by the two concentric circles.

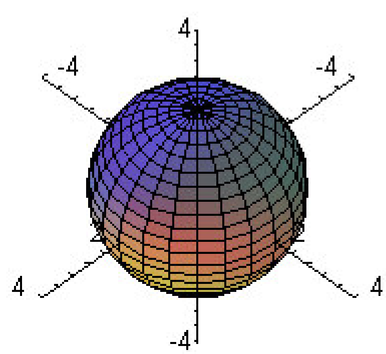
#2. Describe the domain of the following function:

$$f(x,y,z)=\frac{2x+3y+z}{4-x^2-y^2-z^2}$$

$$4-x^2-y^2-z^2\neq 0$$

$$x^2+y^2+z^2\neq 4$$

The domain is the set of all ordered triples which do not lie on the sphere of radius 2 centered at the origin.



#3. Find the limit or show it does not exist. State your reasons for nonexistence.

A.

$$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2};$$

$$\lim_{(x,0)\to(0,0)} \frac{x^2+\sin^2 0}{2x^2+0^2} = \lim_{(x,0)\to(0,0)} \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\lim_{(0,y)\to(0,0)} \frac{0^2+\sin^2 y}{20^2+y^2} = \lim_{(0,y)\to(0,0)} \frac{\sin^2 y}{y^2} = \lim_{(0,y)\to(0,0)} \left(\frac{\sin y}{y}\right)^2 = \left(\lim_{(0,y)\to(0,0)} \frac{\sin y}{y}\right)^2 = 1^2 = 1$$

$$\therefore \textit{Since} \lim_{(x,0)\to(0,0)} f(x,y) \neq \lim_{(0,y)\to(0,0)} f(x,y),$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+\sin^2 y}{2x^2+y^2} \text{ does not exist}$$

B.

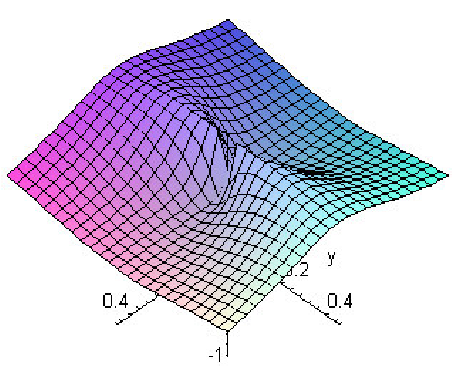
$$\lim_{(x,y)\to(0,0)} \frac{x^2-3y^2}{x^2+2y^2}$$

$$\lim_{(x,0)\to(0,0)} \frac{x^2-30^2}{x^2+20^2} = \lim_{(x,0)\to(0,0)} \frac{x^2}{x^2} = 1$$

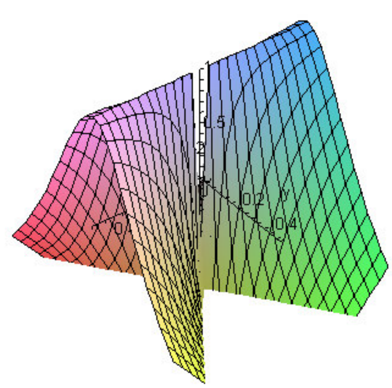
$$\lim_{(0,y)\to(0,0)} \frac{0^2-3y^2}{0^2+2y^2} = \lim_{(0,y)\to(0,0)} \frac{-3y^2}{2y^2} = \frac{-3}{2}$$

$$\therefore \text{ Since } \lim_{(x,0)\to(0,0)} f(x,y) \neq \lim_{(0,y)\to(0,0)} f(x,y),$$

$$\lim_{(x,y)\to(0,0)} \frac{x^2-3y^2}{x^2+2y^2} \text{ does not exist.}$$



A Graph

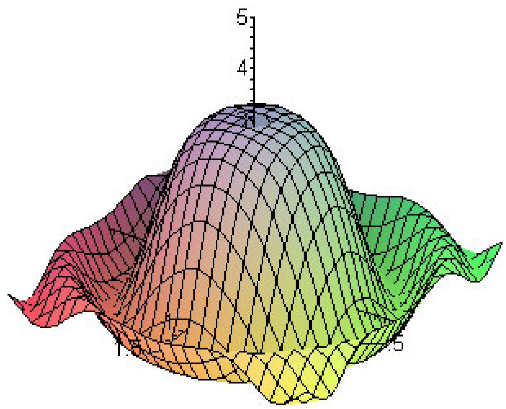


B Graph

C. Use polar coordinates to compute this one: ($x = r\cos\theta; y = r\sin\theta$ $(x,y) \to (0,0)$ *iff* $r \to 0$).

$$\lim_{(x,y)\to(0,0)} \frac{\sin\left(3x^2+3y^2\right)}{x^2+y^2} \to \lim_{r\to 0} \frac{\sin\left(3r^2\left(\cos^2\theta+\sin^2\theta\right)\right)}{r^2\left(\cos^2\theta+\sin^2\theta\right)} = \lim_{r\to 0} \frac{\sin\left(3r^2\right)}{r^2}$$

$$= \lim_{r\to 0} \frac{\cos\left(3r^2\right)6r}{2r} = 3$$



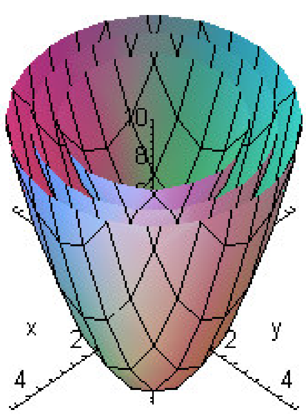
#4. Graph the level surfaces of $F(x,y,z) = x^2 + y^2 - z$ for the values K, where K = 4, 0, and - 4.

$F(x,y,z) = x^2 + y^2 - z$

$z = x^2 + y^2 - 4$ **/* paraboloid shifted 4 down**

$z = x^2 + y^2$ **/*paraboloid**

$z = x^2 + y^2 + 4$ **/* paraboloid shifted 4 up**



#5. Draw a contour map for: $f(x,y)=\cos\sqrt{x^2+y^2}$ **.**

$$f(x,y)=\cos\sqrt{x^2+y^2}$$

$$\cos\sqrt{x^2+y^2}=k \Leftrightarrow x^2+y^2=\cos^{-1}(k) \rightarrow -1\le k\le 1$$

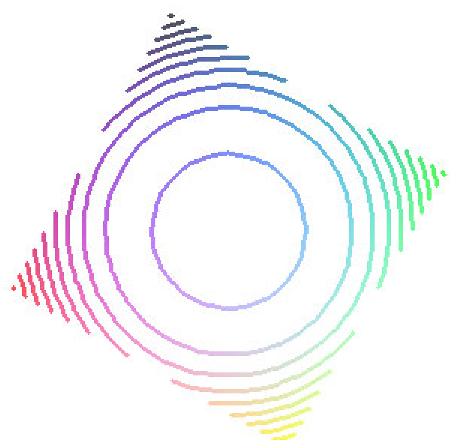
$$\textit{choose } k=-1,0,\frac{\sqrt{2}}{2}$$

$$x^2+y^2=\cos^{-1}(-1)=\pi$$

$$x^2+y^2=\cos^{-1}(0)=\frac{\pi}{2}$$

$$x^2+y^2=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)=\frac{\pi}{4}$$

The contour plot is concentric circles.



#6. Consider the surface:

$$f\left(x,y\right)=10^{y^2-x^2}$$

A. **Determine any restrictions upon the allowed values of K for which the function has level curves. K is positive because:**

$$10^{y^2-x^2}>0 \ \ \forall \ \left(x,y\right)\in \mathbb{R}$$

B. **Pick three convenient values of K and graph the corresponding level curves. Make sure your values sample all possibilities.**

$$f\left(x,y\right)=10^{y^2-x^2}$$

$$\textit{Choose}: \ \ k=\frac{1}{10},10,100$$

$$\frac{1}{10}=10^{y^2-x^2} \Leftrightarrow \log_{10}\left(\frac{1}{10}\right)=\log_{10}\left(10^{y^2-x^2}\right)\Leftrightarrow -1=y^2-x^2$$

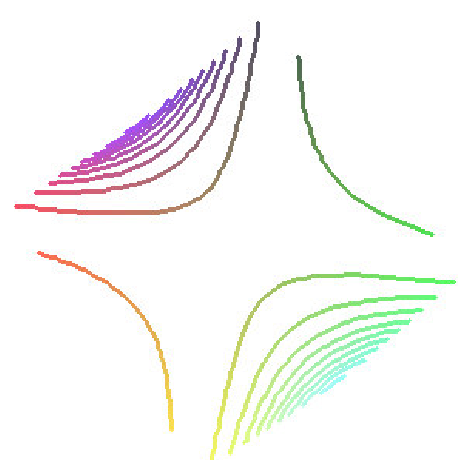
$so, x^2-y^2=1$ **/* hyperbola transverse horizontal**

$$10=10^{y^2-x^2} \Leftrightarrow \log_{10}\left(10\right)=\log_{10}\left(10^{y^2-x^2}\right)\Leftrightarrow 1=y^2-x^2$$

$so, y^2-x^2=1$ **/* hyperbola transverse vertical**

$$100=10^{y^2-x^2} \Leftrightarrow \log_{10}\left(100\right)=\log_{10}\left(10^{y^2-x^2}\right)\Leftrightarrow 2=y^2-x^2$$

$so, y^2-x^2=2$ **/* hyperbola transverse vertical**



Concept Development:

#7. A thin metal rectangular plate, located in the xy-plane, has temperature at point (x, y) on the plate given by $T(x, y) = 5 + 2x^2 + y^2$. The level curves of *T* are called *isothermals* because at all points on an isothermal the temperature is the same. Sketch several isothermals for this plate and be accurate.

$$T(x, y) = 5 + 2x^2 + y^2$$

$$choose: \; k = 9, 21, 41$$

$$5 + 2x^2 + y^2 = 9 \Leftrightarrow 2x^2 + y^2 = 4 \Leftrightarrow \frac{x^2}{2} + \frac{y^2}{4} = 1$$

$$5 + 2x^2 + y^2 = 21 \Leftrightarrow 2x^2 + y^2 = 16 \Leftrightarrow \frac{x^2}{8} + \frac{y^2}{16} = 1$$

$$5 + 2x^2 + y^2 = 41 \Leftrightarrow 2x^2 + y^2 = 36 \Leftrightarrow \frac{x^2}{18} + \frac{y^2}{36} = 1$$

$$\therefore nested \; ellipses$$

#8. In the clean and jerk weightlifting competition a weightlifter in the heavyweight class who weighs 110 kg lifts 210 kg. In the flyweight class, a person who weighs 50 kg lifts 130 kg. How can we compare these feats of strength? Who actually is the superior lifter? Several different formulas have been developed for handicapping lifts. Let w_l be the lifted weight (in kg) and w_b be the body weight of the lifter (in kg). Consider the following 4 proposed handicapping formulas.

A. **Used in ABC's Superstars competition:** $h = w_l - w_b$

$$h_{HW} = 210 - 110 = 100$$

$$h_{FW} = 130 - 50 = 80$$

B. **Austin formula:** $h = \dfrac{w_l}{w_b^{\frac{3}{4}}}$

$$h_{HW} = \frac{210}{110^{\frac{3}{4}}} \approx 6.18$$

$$h_{FW} = \frac{130}{50^{\frac{3}{4}}} \approx 6.91$$

C. **Classical formula;** $h = \dfrac{w_l}{w_b^{\frac{2}{3}}}$

$$h_{HW} = \frac{210}{110^{\frac{2}{3}}} \approx 9.15$$

$$h_{FW} = \frac{130}{50^{\frac{2}{3}}} \approx 9.58$$

D. **O'Carroll formula:** $h = \dfrac{w_l}{(w_b - 35)^{\frac{1}{3}}}$

$$h_{HW} = \frac{210}{(65)^{\frac{1}{3}}} \approx 52.23$$

$$h_{FW} = \frac{130}{(15)^{\frac{1}{3}}} \approx 52.71$$

Use these formulas to determine whether the heavyweight or the flyweight is the superior lifter.

In each case the fly-weight has a greater handicap so he/she is the superior lifter.

Computer Graphics & Computations.
#9. Discuss the continuity of the function and evaluate the limit along the suggested paths. Does the limit exist? Why or why not? Include with your analysis a computer generated graph.
A.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \quad paths \;\; y=0 \;\&\; y=x$$

$$\lim_{(x,0)\to(0,0)} \frac{xy}{x^2+y^2} = \lim_{(x,0)\to(0,0)} \frac{0}{x^2} = 0 \;\; \forall x \neq 0$$

$$\lim_{(x,x)\to(0,0)} \frac{x^2}{x^2+x^2} = \lim_{(x,x)\to(0,0)} \frac{x^2}{2x^2} = \frac{1}{2}$$

$$\therefore \text{ Since } \lim_{(x,0)\to(0,0)} \frac{xy}{x^2+y^2} \neq \lim_{(x,x)\to(0,0)} \frac{x^2}{x^2+x^2},$$

$$\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2} \text{ does not exist}$$

B.

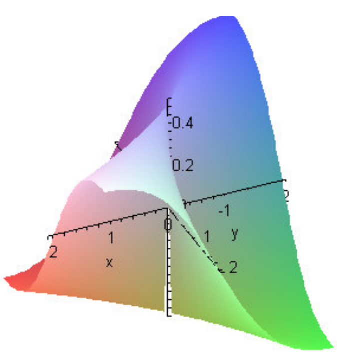
$$\lim_{(x,y)\to(0,0)} \frac{2x-y^2}{2x^2+y} \quad paths \;\; y=0 \;\&\; y=x$$

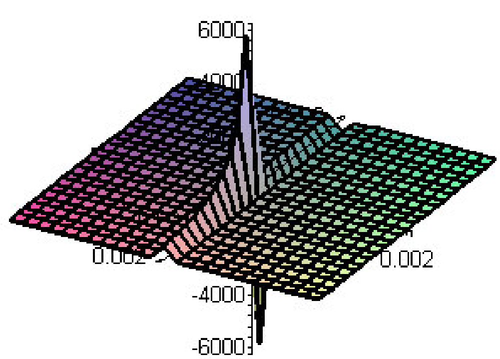
$$\lim_{(x,0)\to(0,0)} \frac{2x}{2x^2} = \lim_{(x,0)\to(0,0)} \frac{1}{x} \to \pm\infty$$

$$\lim_{(x,x)\to(0,0)} \frac{2x-x^2}{2x^2+x} = \lim_{(x,x)\to(0,0)} \frac{2-x}{2x+1} = 2$$

$$\therefore \text{ Since } \lim_{(x,0)\to(0,0)} f(x,y) \neq \lim_{(x,x)\to(0,0)} f(x,y),$$

$$\lim_{(x,y)\to(0,0)} \frac{2x-y^2}{2x^2+y} \text{ does not exist}$$





Special Project

When my father studied to become a tool & die maker in the fifties people used clay to visualize how planes would intersect cylinders and other various shapes. They could cut the clay at angles and then see the resulting cross sections. The tool they used for cutting the clay can still be bought today and simply consists of a wire strung between two rigid rods. Two types available appear below.

Today, we have the advantage of powerful three dimensional software packages but the visualization process for students is still extremely important. Building realistic "models" both in a concrete or virtual world is a skill to be mastered by those serious about their mathematical and engineering pursuits. Consider a simple slicing.

A cylinder cut by a plane making a 45 degree angle with the horizontal.

Complete the following.

1. **Build a physical model of the scenario. An old toilet paper roll or some wood stock would do fine. I hope to get some great models!**
2. **Construct a "virtual" model by doing the following**

- **Determine the equation for a cylinder with radius 1.**
- **Determine the equation of the plane which cuts the cylinder at a 45°and contains the point:** $\left(\frac{\sqrt{2}}{2},\frac{\sqrt{2}}{2},0\right)$ **and has a nice traces in octant (x^+,y^+).**
- **Graph the cylinder, slanted plane, and the xy-plane all on one graph.**
- **Determine the space curve which represents the intersection of the plane and the cylinder.**
- **Determine the length of one trace of the space curve.**
- **EC. Determine the volume and surface area (entire solid) of the wedge cut out by the xy-plane, the slanted plane, and the cylinder. Come by my office for a visual.**