Mat 241 Homework Set 3 – Due ______________

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Directions: **Show *all algebraic* steps neatly and concisely using *proper mathematical symbolism*. When graphs and technology are to be implemented, do so appropriately.**

Mechanics:

#1. Determine the limit: $\lim_{t\to 1}\left\langle \frac{3}{t^2},\frac{\ln t}{t^2-1},e^t\sin\left(\frac{\pi}{3}t\right)\right\rangle$

#2. A particle travels along the path $\vec{r}(t)=\left\langle 2t,\frac{7}{2}t^2\right\rangle \quad 0\le t\le\infty$ **seconds.**

A. **On a separate graph hand sketch a trace of the vector function with t = 0, 1, 2, and 4 seconds.**

B. **Determine the position vector and the unit tangent vector** $\mathbf{T}(\mathbf{t})=\frac{\vec{\mathbf{r}}'(\mathbf{t})}{\left\|\vec{\mathbf{r}}'(\mathbf{t})\right\|}$ **at t = 1 second and hand draw both of these vectors on the graph in part B.**

C. **Eliminate the parameter, t, obtaining a function of the form y=f(x) and hand graph this function over the domain [0,4]. Is this the same graph as in part A?**

#3. Suppose that the velocity of a particle is modeled by the vector function $\vec{v}(t)=\left\langle \frac{2t^2}{16-3t^2},te^{3t},\sin^3(4t)\right\rangle$**. Find the position vector function** $\vec{r}(t)$ **and the acceleration vector function** $\vec{a}(t)$**.**

#4. Suppose the trajectories of two objects are modeled by:

$$\vec{r}_1(t)=\left\langle t^2,7t-12,t^2\right\rangle;\ \vec{r}_2(t)=\left\langle 4t-3,t^2,5t-6\right\rangle \quad t\ge 0$$

A. **Determine if these particles ever collide.**
B. **If their paths do cross, how many times does it occur and when?**

#5. Determine the *tangent line* to $\vec{r}(t)=\left\langle \sin\left(\pi e^t\right),\cos\left(\pi e^t\right),e^t\right\rangle$ **at t = 0.**

<u>Concept Development & Graphics:</u>

#6. Compute the tangent vectors and *unit* tangent vectors to the curves:

$$\vec{r}(t) = \left\langle \cos t, \sin t, \frac{\sqrt{5}}{3} \right\rangle \ \ \& \ \ \vec{s}(t) = \left\langle \cos t, \sin t, -\frac{\sqrt{5}}{3} \right\rangle$$

A. **Algebraically show that these two vectors form the intersection of the ellipsoid $\frac{4}{9}x^2 + \frac{4}{9}y^2 + z^2 = 1$ and the cylinder $x^2 + y^2 = 1$.**
B. **Use Maple or some other software package to graph the two surfaces on a grid (see helpful code). Make sure they are both on the same graph.**
C. **Compute the tangent vectors and *unit* tangent vectors to the curves. See page 858 example 1. Why do these two vectors have zero $\vec{k}$ components?**
D. **What angular change in direction is measured by $T'(t)$ for both of these curves?**

#7. I claim that $T'(t) \perp T(t)$. Verify this result for the case when

$$\vec{r}(t) = t^3\vec{i} + t^6\vec{j}$$

#8. There are several space curves that are quite interesting. Use a plotting software package like Maple to plot the 4 curves listed below.

A. **Elliptical Helix:** $2\cos t\vec{i} + 2\sin t\vec{j} + t\vec{k}$

B. **Conical Helix:** $\vec{r}(t) = \langle t\cos t, t\sin t, t^2 \rangle$

C. $\vec{r}(t) = \langle \sin t, (1-\cos t)^3, \sin(1-\cos t) \rangle$

D. $\vec{r}(t) = \langle t\sin t\cos t, t\sin^2 t, t\sin t \rangle$ let $0 \le t \le 8\pi$

To plot surfaces use this Maple code: Example:

A. **Graph the following two surfaces: The ellipsoid:** $\frac{x^2}{16}+\frac{y^2}{4}+\frac{z^2}{4}=1$

   and the cylinder: $x^2+y^2=4$

B. **Determine parametric equations for the space curve of intersection. Then plot the surfaces and the space curve of intersection on the same graph**

Let $x=2\cos(t)$ **and** $y=2\sin(t)\rightarrow note: x^2+y^2=4$

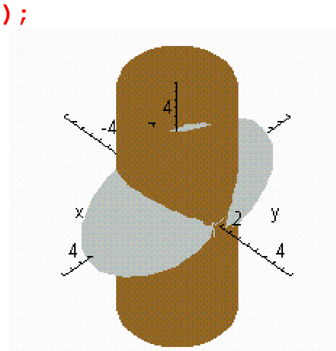
Using the above choice for x & y, find the z(t) function(s) from the ellipsoid.

$$\frac{x^2}{16}+\frac{y^2}{4}+\frac{z^2}{4}=1\Leftrightarrow\frac{4\cos^2 t}{16}+\frac{4\sin^2 t}{4}+\frac{z^2}{4}=1\Leftrightarrow z^2=4-\cos^2 t-4\sin^2 t$$

$$\therefore z(t)=\sqrt{4-\cos^2 t-4\sin^2 t}\ \text{ and } z(t)=-\sqrt{4-\cos^2 t-4\sin^2 t}$$

Solution to A.

```
> restart:with(plots):with(plottools):
Warning, the name changecoords has been redefined
Warning, the assigned name arrow now has a global binding
> a:=implicitplot3d(x^2/(16)+y^2/4+z^2/4=1, x=-5..5,y=-5..5,z=-5..5,numpoints=10000,color=gray,style = patchnogrid):
> b:=implicitplot3d(x^2+y^2=4, x=-5..5,y=-5..5,z=-5..5,numpoints=10000,color=sienna,style=patchnogrid):
> display(a,b,axes=normal);
```



Solution to B.

```
> c:=spacecurve([2*cos(t),2*sin(t), sqrt(4-(cos(t))^2-4*(sin(t))^2)],t=0..2*Pi,thickness=3,color=black):
d:=spacecurve([2*cos(t),2*sin(t), -sqrt(4-(cos(t))^2-4*(sin(t))^2)],t=0..2*Pi,thickness=3,color=black):
> display(a,b,c,d,axes=normal);
```