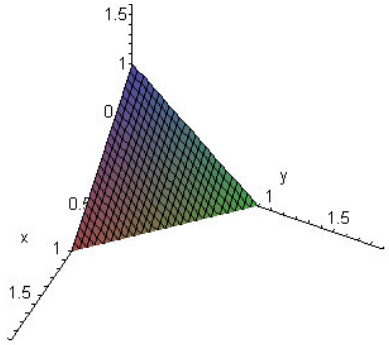
Surface Integrals 16.7 Part 1
non-parameterized surfaces

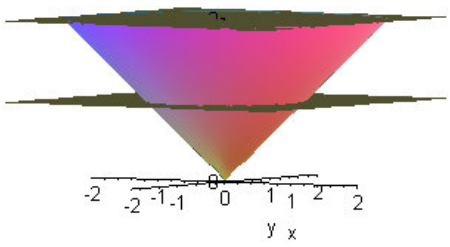
$$\iint_S f(x,y,z)dS = \iint_D f(x,y,g(x,y))\sqrt{\left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2 + 1}\,dA$$

Surface: $g(x,y) = z$

Example: Compute $\iint_S xz\,dS$ if the surface is the part of the plane given by: $x + y + z = 1$ in the first octant.



Example: Compute $\iint_S y^2 z^2\,dS$ where S is the part of the cone $z = \sqrt{x^2 + y^2}$ between the planes z = 1 and z = 2.

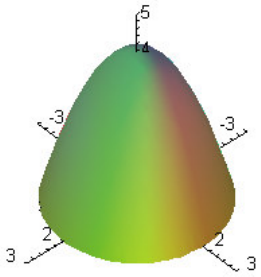


Surface Integral of a Vector Field – "Flux Integral"

$$\iint_S \vec{F}\cdot d\vec{S} = \iint_S \vec{F}\cdot \vec{n}\,dS = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + 1\right)dA$$

where $\vec{n}$ is an upward normal.

Example: Compute $\iint_S \vec{F}\cdot d\vec{S}$. Let $\vec{F}(x,y,z) = xy\vec{i} + yz\vec{j} + zx\vec{k}$. Bound the paraboloid $z = 4 - x^2 - y^2$ by the xy – plane.



Example: A fluid with density 1200 flows with a velocity vector of $\vec{v}(x,y,z) = y\vec{i} + \vec{j} + z\vec{k}$. Find the rate of flow upward through the paraboloid $z = 9 - \frac{1}{4}(x^2 + y^2)$; where $x^2 + y^2 \le 36$