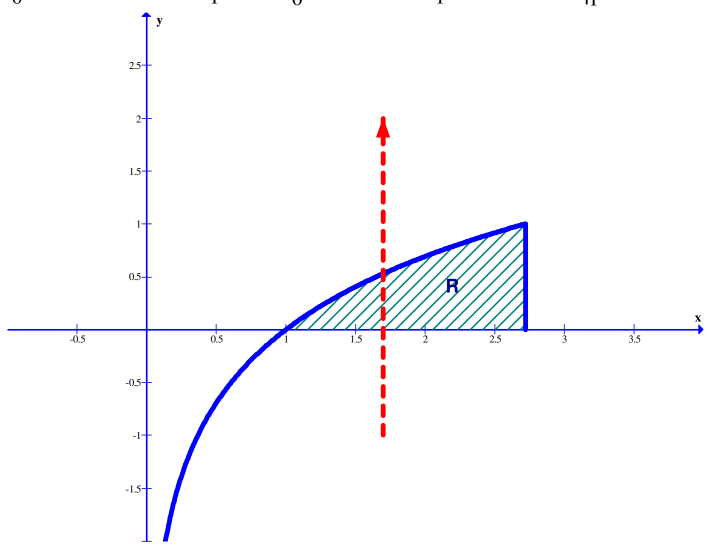
Mat 241 Chapter 15 Test
2009 **Name ________________________________**
Professor Schultz

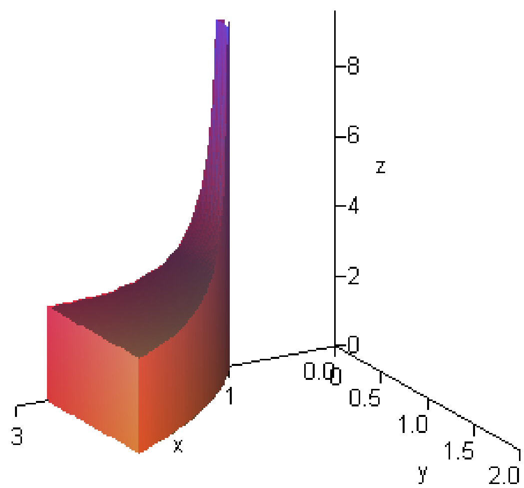
#1. Compute the integral by reversing the order of integration (3 pts)

$$\int_0^1 \int_{e^y}^{e} \frac{x}{\ln x} dxdy = \int_{y=0}^{y=1} \int_{x=e^y}^{x=e} \frac{x}{\ln x} dxdy$$

$$x = e^y \Leftrightarrow y = \ln x$$

$$\to \int_1^e \int_0^{\ln x} \frac{x}{\ln x} dydx = \int_1^e \left. \frac{xy}{\ln x} \right|_0^{\ln x} dydx = \int_1^e xdx = \left. \frac{x^2}{2} \right|_1^e = \frac{1}{2}\left(e^2 - 1\right)$$





#2. Compute the exact value of the integral. **(3 pts)**

$$\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_{1}^{4-x^2-y^2}\frac{1}{z^2}dzdydx$$

$$\int_{-\sqrt{3}}^{\sqrt{3}}\int_{-\sqrt{3-x^2}}^{\sqrt{3-x^2}}\int_{1}^{4-x^2-y^2}\frac{1}{z^2}dzdydx=\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}\int_{1}^{4-r^2}\frac{1}{z^2}rdzdrd\theta=-\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}\frac{1}{z}\Big|_{1}^{4-r^2}rdrd\theta$$

$$=-\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}\left(\frac{r}{4-r^2}-r\right)drd\theta=$$

$$-\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}\left(\frac{r}{4-r^2}\right)drd\theta+\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}(r)drd\theta$$

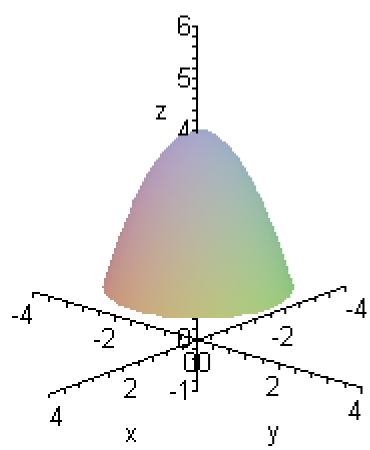
$$=\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}(r)drd\theta-\int_{0}^{2\pi}\int_{0}^{\sqrt{3}}\left(\frac{r}{4-r^2}\right)drd\theta$$

$$=\int_{0}^{2\pi}\frac{r^2}{2}\Big|_{0}^{\sqrt{3}}d\theta+\frac{1}{2}\int_{0}^{2\pi}\int_{4}^{1}\frac{1}{u}dud\theta$$

$$=\int_{0}^{2\pi}\frac{3}{2}d\theta-\frac{1}{2}\int_{0}^{2\pi}\int_{1}^{4}\frac{1}{u}dud\theta$$

$$=3\pi-\frac{1}{2}\int_{0}^{2\pi}\ln|u|\Big|_{1}^{4}d\theta=3\pi-\frac{1}{2}\int_{0}^{2\pi}(\ln 4-\ln 1)d\theta=3\pi-\ln 2\int_{0}^{2\pi}d\theta$$

$$=3\pi-2\pi\ln 2$$

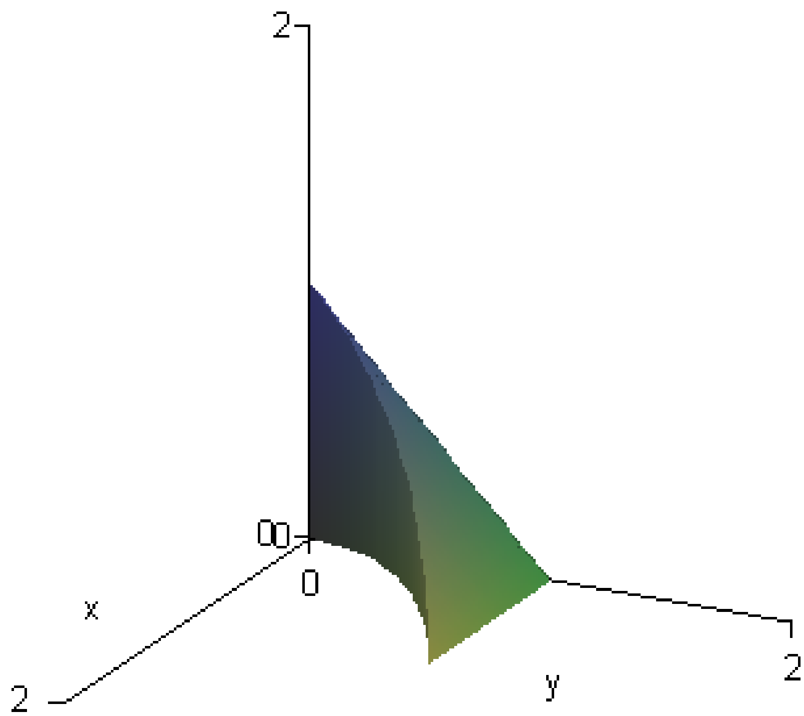


#3. Find the exact volume of the solid formed in the first octant bounded above by the plane $z = 1 - y$, below by the xy – plane, the plane y = 0, and by the surface $y = \sqrt{x}$. (3 pts)

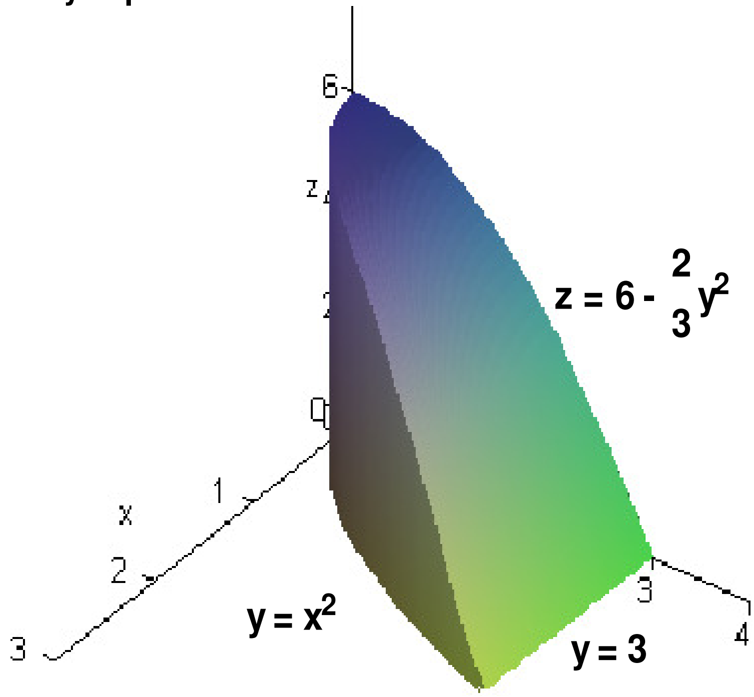
$$V = \int_0^1 \int_{\sqrt{x}}^1 (1-y)\,dy\,dx \quad \text{or} \quad V = \int_0^1 \int_0^{1-y} y^2\,dz\,dy \quad (easier)$$

$$V = \int_0^1 \int_0^{1-y} y^2\,dz\,dy = \int_0^1 y^2 z\Big|_0^{1-y} dy = \int_0^1 \left[y^2(1-y)\right]dy = \int_0^1 \left(y^2 - y^3\right)dy$$

$$= \left(\frac{y^3}{3} - \frac{y^4}{4}\right)\Bigg|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

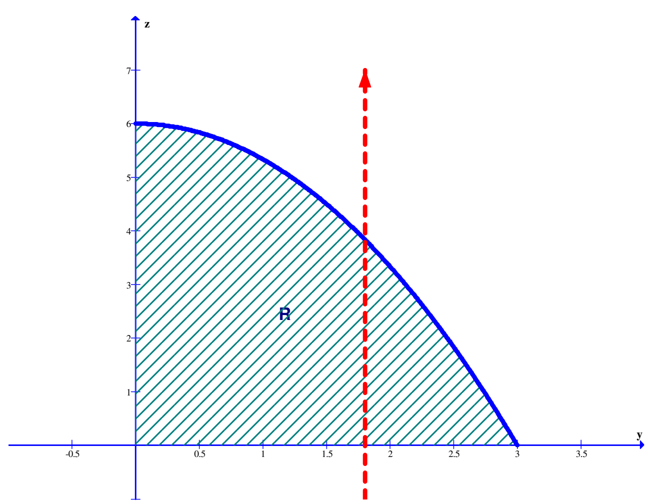


#4. For the pictured solid assume that the density is proportional to the distance to the yz - plane.



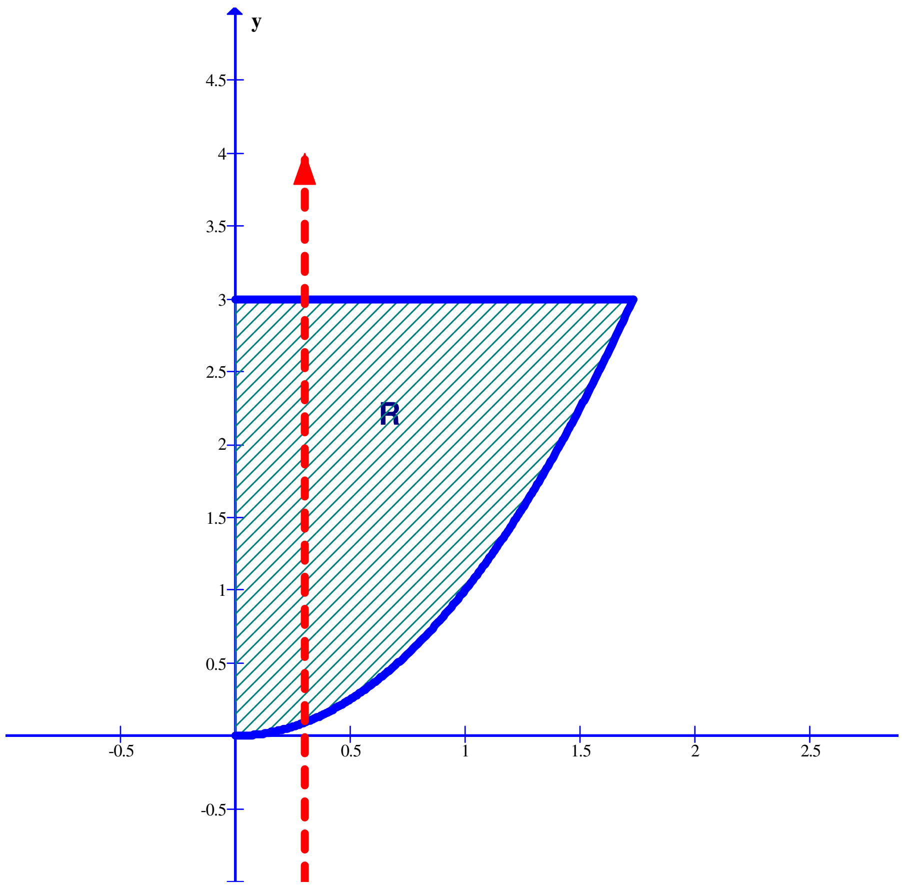
A. Set – up ONLY! An integral to compute the mass of the solid in the dxdzdy order. ALSO sketch the plane projection. **(2 pts)**

$$mass = \int_0^3 \int_0^{6-\frac{2}{3}y^2} \int_0^{\sqrt{y}} kx\,dx\,dz\,dy$$



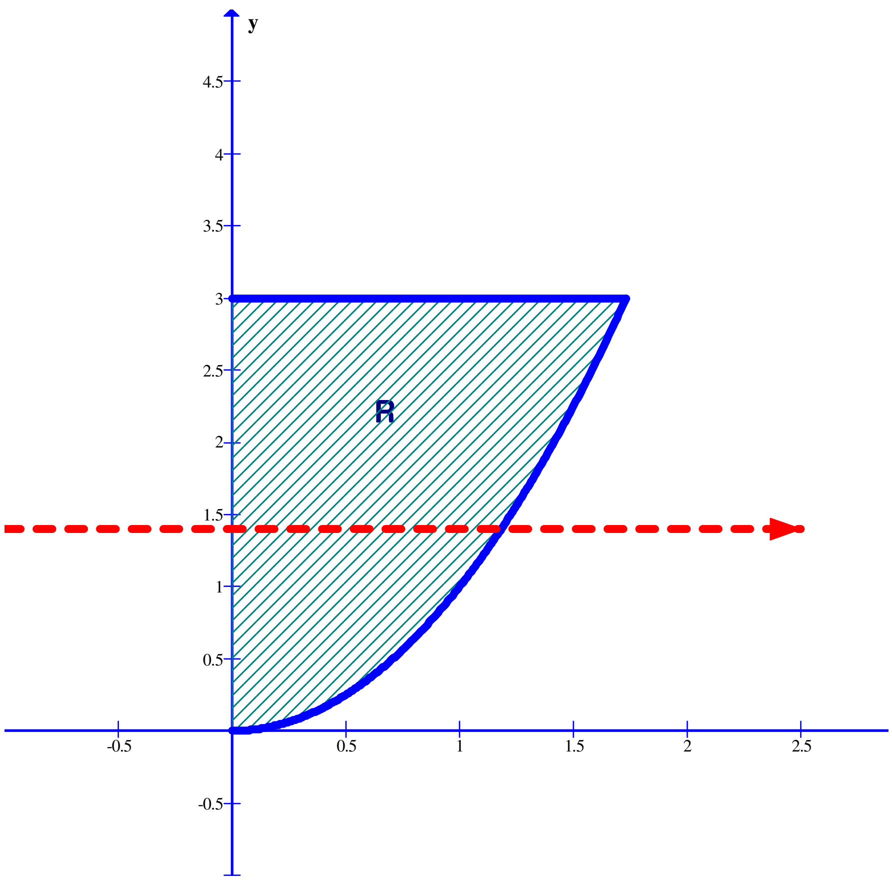
B. Set – up ONLY! An integral to find M_{xy} in the dzdydx order. ALSO sketch the plane projection. (2 pts)

$$M_{xy} = \int_0^{\sqrt{3}} \int_{x^2}^{3} \int_0^{6-\frac{2}{3}y^2} kxz\,dz\,dy\,dx$$



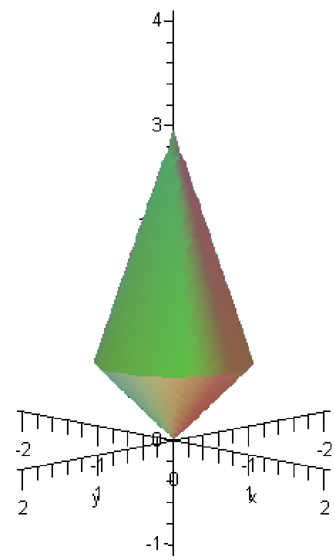
C. Set – up ONLY! An integral to find I_x in the dzdxdy order. ALSO sketch the plane projection. **(2 pts)**

$$I_x = \int_0^3 \int_0^{\sqrt{y}} \int_0^{6-\frac{2}{3}y^2} kx\left(y^2+z^2\right)dzdxdy$$



#5. A metal plug is designed by using the intersection of two cones with the given equations shown below. Determine the *exact volume* of the plug using either a double or triple integral. (5 pts)

$$\sqrt{x^2+y^2}+\frac{z}{3}=1 \qquad\qquad z=\sqrt{x^2+y^2}$$



$$\sqrt{x^2+y^2}+\frac{z}{3}=1;\quad z=\sqrt{x^2+y^2}\to r$$

$$\frac{z}{3}=1-r \Leftrightarrow z=3-3r;\quad r=3-3r \Leftrightarrow r=\frac{3}{4}$$

$$V=\int_0^{2\pi}\int_0^{\frac{3}{4}}[3-3r-r]r\,dr\,d\theta=\int_0^{2\pi}\int_0^{\frac{3}{4}}\left[3r-4r^2\right]r\,dr\,d\theta=\int_0^{2\pi}\left(\frac{3r^2}{2}-\frac{4r^3}{3}\right)\Bigg|_0^{\frac{3}{4}}$$

$$=\int_0^{2\pi}\left(\frac{27}{32}-\frac{4}{3}\cdot\frac{27}{64}\right)d\theta=\int_0^{2\pi}\left(\frac{27}{32}-\frac{9}{16}\right)d\theta=\int_0^{2\pi}\left(\frac{27}{32}-\frac{18}{32}\right)d\theta=\int_0^{2\pi}\left(\frac{9}{32}\right)d\theta$$

$$=\frac{9}{32}\theta\Big|_0^{2\pi}=\frac{18\pi}{32}=\frac{9\pi}{16}$$

Hey, we can check old school by: $V_{cone}=\frac{1}{3}\pi r^2 h$.

$$V=\frac{1}{3}\pi\left(\frac{3}{4}\right)^2\left(\frac{3}{4}\right)+\frac{1}{3}\pi\left(\frac{3}{4}\right)^2\left(3-\frac{3}{4}\right)=\frac{9\pi}{64}+\frac{1}{3}\pi\left(\frac{3}{4}\right)^2\left(\frac{9}{4}\right)=\frac{9\pi}{64}+\frac{27\pi}{64}=\frac{36\pi}{64}=\frac{9\pi}{16}$$

I guess there is order in the universe.