Section 13.2 example.

Find the parametric equations for the tangent line to the curve at the specified point. Specify the tangent and unit tangent vectors.

$$\vec{r}(t)=\left\langle \ln t, 2\sqrt{t}, t^2 \right\rangle; P(0,2,1)$$

solution

Our space curve takes on the point P when t = 1.

$\vec{r}(t)=\left\langle \ln t, 2\sqrt{t}, t^2 \right\rangle; P(0,2,1)$

$\vec{r}'(t)=\left\langle \frac{1}{t}, \frac{1}{\sqrt{t}}, 2t \right\rangle$ **/* this is the tangent vector for all permissable t.**

$\vec{r}'(1)=\langle 1,1,2 \rangle$ **/* this is the specific tangent vector for t = 1.**

$x=0+1t, y=2+1t, z=1+2t$

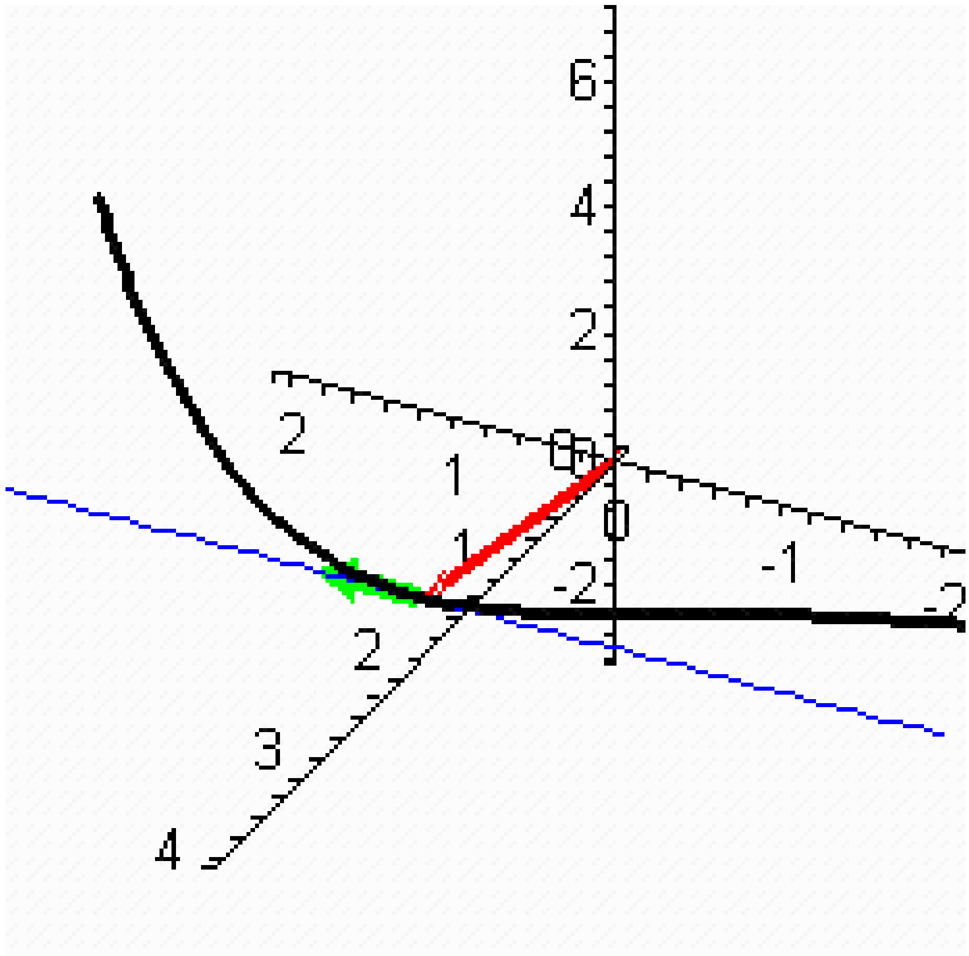
$\therefore x=t, y=2+t, z=1+2t$ **/* these are the parametric eq. for the tangent line.**

$\vec{\mathbf{T}}(\mathbf{t})=\frac{\vec{r}'(t)}{\left|\vec{r}'(t)\right|}$ **/* this is the definition of a unit tangent vector.**

$\left|\vec{r}'(t)\right|=\sqrt{\left(\frac{1}{t}\right)^2+\left(\frac{1}{\sqrt{t}}\right)^2+(2t)^2}=\sqrt{\frac{1}{t^2}+\frac{1}{t}+4t^2}$ **/*norm of all tangent vectors**

$\therefore \vec{\mathbf{T}}(\mathbf{1})=\frac{\vec{r}'(1)}{\left|\vec{r}'(1)\right|}=\frac{1}{\sqrt{6}}\langle 1,1,2\rangle=\left\langle \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}} \right\rangle$ **/*unit tangent vector**

```
> restart:with(plots):with(plottools):
Warning, the name changecoords has been redefined
Warning, the assigned name arrow now has a global binding
> curv:=spacecurve([ln(t),2*sqrt(t),t^2],t=0..3,thickness=2,color=black):
> positionvector:= arrow([0,0,0],[0,2,1], .2, .4, .1, color=red,thickness=3):
tanline:=spacecurve([t,2+t,1+2*t],t=-2..2,color=blue,thickness=1):
unittan:=arrow([0,2,1],[1/sqrt(6),2+1/sqrt(6),1+2/sqrt(6)],.2, .4, .4,
color=green,thickness=3):
>
display(curv,positionvector,unittan,tanline,axes=normal,orientation=[117,59]);
```



6
4
2
2
1
0
0
-1
-2
1
-2
2
3
4