15

My Family Taught Me This Way

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I REMEMBER the first time I saw subtraction done by using the algorithm taught in American schools. I was reading a paper on the typical mistakes children make when learning multidigit subtraction, and I felt like one of those children, trying to make sense of the numbers in front of me. I wondered what those crossed-out numbers were and what the author of the paper meant by *ignored borrows.* I had to read the paper three times before I realized that my way of doing subtraction was very different from that of the author of the paper and of the children making those typical mistakes. There I was, a reasonably intelligent adult woman, and yet I could make sense of the American subtraction algorithm only with the aid of a paper. Since then I have met many other people who have shown an equal lack of understanding when faced with multidigit subtraction as solved by American-schooled children. Those people and I did not necessarily share a language or a culture, but we did share a common trait: we had all been schooled in Europe or in South America.

I have spent the last three years working on a grades K–3 mathematics research project in bilingual inner-city schools in which 88 percent of the children have a Latino background (see Fuson, Feingold, and Cuevas [1996] for details). Our approach is to allow children to use different methods with the support of base-ten materials for understanding. However, we have worked with teachers who teach the standard U.S. algorithm and have seen the confusions that result when parents that were schooled in other countries teach their children their "add tens to both" method (a method found in most European and Latin countries, among others). Because parents will always try to help their children with mathematics as they know it, it is helpful for teachers to understand the European-Latino algorithm (E-L algorithm), the errors that children make with it, and the confusions that arise from mixing the U.S. and the E-L algorithms.

THE ALGORITHMS

A common U.S. algorithm for subtraction is demonstrated in figure 15.1: Borrow one ten to go with the ones, that is, regroup the top number.

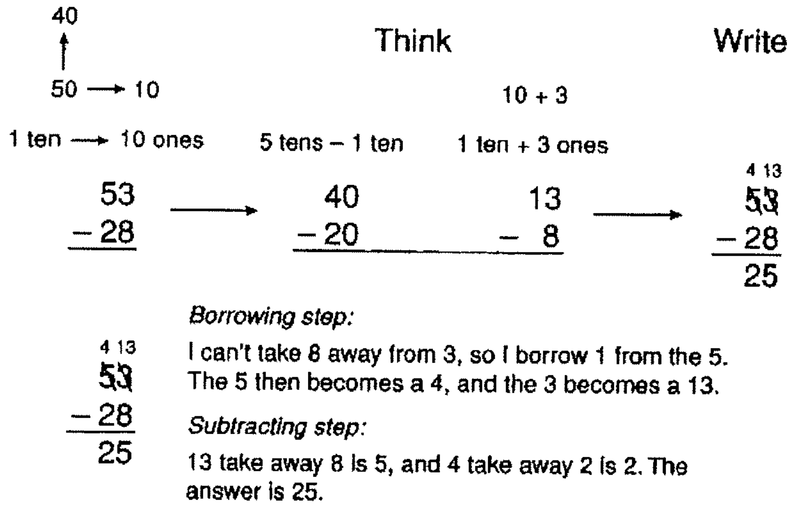

Fig. 15.1. The U.S. subtraction algorithm

The E-L algorithm for subtraction is shown in figure 15.2: Add a ten to both numbers. This algorithm, which is also known as the *equal additions method,* is found in many countries, including most European and South American countries. The E-L algorithm relies on the fact that the result of subtracting 28 from 53 is the same as that of subtracting 38 from 63. To benefit from this mathematical fact, both numbers are changed equally by adding a ten to each number. First add 1 ten to the ones in the top number (the minuend), so 3 becomes 13. Then add the other ten to the tens in the bottom number (the subtrahend), so 2 tens becomes 3 tens. The actual subtracting is usually done as counting up (i.e., rather than "13 take away 8 is 5," the child says or thinks "from 8 to 13 is 5"). The adding of a ten to the subtrahend is expressed by the phrase "I carry 1," which is the same wording used in the addition algorithm when a 1 (actually a ten) is carried over. For users of the E-L algorithm, the similarity of expressions opens the door to the potential confusion of addition and subtraction. Typically one thinks of putting a 1 in front of the 3 to make 13 and then carrying 1 into the tens column in the subtrahend instead of thinking of adding a ten. As with the U.S. algorithm, most parents do not understand why the E-L algorithm works or that it really adds ten to *each* multidigit number.

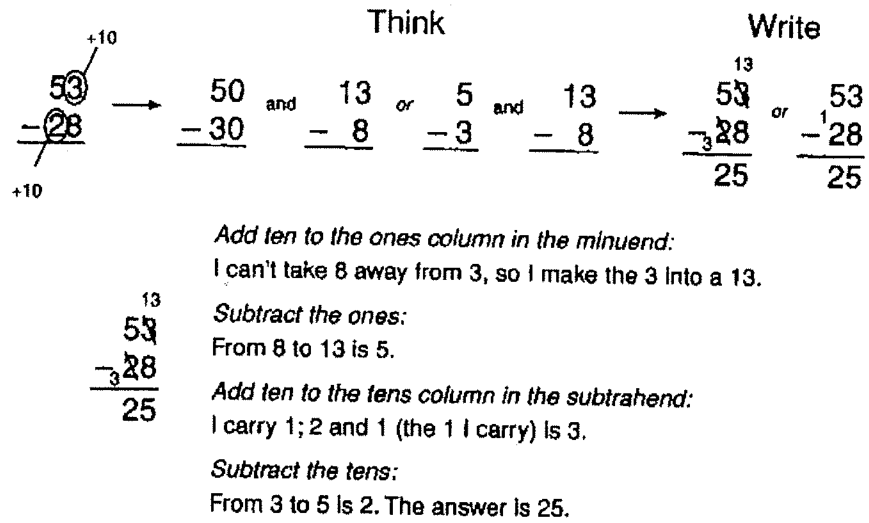

Fig. 15.2. The European-Latino subtraction algorithm

TYPICAL ERRORS AND CONFLICTS

Both algorithms may be used by students in a classroom even if a teacher does not teach them because they may be taught in the homes. Because some parents emphasize working mentally, teachers may not see marks on paper that can be used to diagnose errors that students may be making. Asking students to explain their methods can help the teacher uncover the reasons for the errors that do arise.

A child who has been exposed to both the U.S and the E-L algorithms may make mistakes that stem from taking characteristic elements from both. The phrase "my family taught me this way" is often heard in the classroom as an explanation for a procedure that students realize was not taught in class. Teachers need to understand the reason for the potential mistakes so that they can provide help wherever it is needed.

The two most common mistakes that result from mixing the two subtraction algorithms are shown in figure 15.3. Some children begin with the U.S. borrowing algorithm, then switch algorithms during the move to the tens column. They "carry 1" but in doing so shift to some form of addition. In version 1 they shift completely to addition and end up subtracting in the ones column and adding in the tens column. In version 2 they do not shift to adding the tens column but add the ten to the minuend instead of to the subtrahend before subtracting in the tens column. (These examples are not the work of any one child but represent recurring mistakes that we have found among lower-achieving children of Latino background.)

Mixed U.S.-E/L Version 1
(the 1, 4, and 13 may or may not be written)

```
    1
  4 13
   53
 − 28
   75
```

Borrow to subtract ones (U.S.):
I can't take 8 away from 3, so I borrow 1 from the 5. The 5 then becomes a 4 and the 3 becomes a 13.

Subtract ones (U.S.):
13 take away 8 is 5.

Carry the 1 (E/L):
I carry 1.

E/L error: Shift to addition so the 1 is added to the tens column in the minuend instead of in the subtrahead, and the entire tens column is then added:
The 1 I carry and 4 is 5 and 2 is 7. The answer is 75.

Mixed U.S.-E/L Version 2
(the 1, 4, 5, and 13 may or may not be written)

```
 1 5
   4 13
   53
 − 28
   35
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Borrow to subtract ones (U.S.):
I can't take 8 away from 3, so I borrow 1 from the 5. The 5 then becomes a 4 and the 3 becomes a 13.

Subtract ones (U.S.):
13 take away 8 is 5.

Carry the 1 (E/L):
I carry 1.

E/L error: Shift to addition so the 1 is added to the tens column in the minuend instead of in the subtrahead, and then the subtraction is performed in the tens column:
The 1 I carry and 4 is 5; 5 take away 2 is 3. The answer is 35.

Fig. 15.3. Common errors resulting from mixing the U.S. and the European-Latino subtraction algorithms

The result of the error in version 2 is the same as that obtained if the child had done the borrowing in the ones column but failed to follow through in the tens column. If the student does the procedure mentally, the teacher has no way of knowing if the child forgot to do the borrowing, did the borrowing and then ignored it, or did something else (as in this example). A correction like "You forgot to borrow" would not be of use because the child did not forget. A teacher needs to ask the child how she or he did the subtraction to discover the source of the error.

Although all the foregoing errors are certainly possible—and to some degree common—among lower-achieving children who use the E-L algorithm in American schools, the typical progression is for them to abandon the algorithm they were taught at home and become proficient in the one taught at school if a standard algorithm is taught. However, some children remain confused for a long time. A better understanding by both teachers and parents of the source of the confusion could and should lead to an early correction of the procedure for solving multidigit subtraction.

CONCLUSIONS

In a classroom where mathematics is taught with an emphasis on understanding, the U.S. algorithm shows a clear advantage over the E-L algorithm because teachers feel it can be explained meaningfully. A child can be helped to understand that borrowing is actually a regrouping of 53 (5 tens and 3 ones) into 40 + 13 (4 tens and 13 ones). In the current worldwide trend of teaching mathematics for understanding, the U.S. algorithm may be taught in European and Latin American schools in the near future. (In some teachers colleges in Spain, the U.S. algorithm is already being taught to preservice teachers as a preferred alternative to the traditional E-L method.) When and if that happens, teachers from European and Latin American countries will benefit from understanding the two different algorithms and the errors associated with the unexpected mixing of the algorithms that result when children get conflicting messages at school and home.

In a country like the United States where the student population can be so diverse, it is important that teachers find out what knowledge students from all backgrounds bring from their cultures and their homes and *accept it* rather than try to impose their own. Understanding how other algorithms work can give teachers the necessary resources to uncover the reason for certain mistakes that at first glance are unexplainable. Allowing alternative algorithms into the classroom would give voice and value to the knowledge coming from students' homes and has the potential to introduce other mathematical ideas that can be pursued.

REFERENCE

Fuson, Karen, Cathy Feingold, and Saúl Cuevas. "Children's Math Worlds: A Teaching/Learning Project to Support Urban Latino Children's Construction of Mathematical Understanding." Paper presented at the 1996 annual meeting of the American Education Research Association, New York, 8–12 April 1996.