In Exercises 1–4, determine whether \( f(x) \) approaches \( \infty \) or \( -\infty \) as \( x \) approaches \(-2\) from the left and from the right.

1. \( f(x) = \frac{1}{(x + 2)^2} \)
2. \( f(x) = \frac{1}{x + 2} \)

\[ y \]

\[ -3 \quad -2 \quad -1 \quad 0 \quad 1 \]

\[ -1 \quad 0 \quad 1 \]

3. \( f(x) = \tan \frac{\pi x}{4} \)
4. \( f(x) = \sec \frac{\pi x}{4} \)

\[ y \]

\[ -6 \quad -3 \quad 0 \quad 3 \quad 6 \]

\[ -3 \quad 0 \quad 3 \]

In Exercises 5–8, determine whether \( f(x) \) approaches \( \infty \) or \( -\infty \) as \( x \) approaches \(-3\) from the left and from the right.

5. \( f(x) = \frac{1}{x^2 - 9} \)
6. \( f(x) = \frac{x}{x^2 - 9} \)
7. \( f(x) = \frac{x}{x^2 - 9} \)
8. \( f(x) = \sec \frac{\pi x}{6} \)

In Exercises 9–24, find the vertical asymptotes (if any) of the function.

9. \( f(x) = \frac{1}{x^2} \)
10. \( f(x) = \frac{2}{x - 3} \)
11. \( f(x) = \frac{x^2 - 2}{x^2 - x - 2} \)
12. \( f(x) = \frac{2 + x}{1 - x} \)
13. \( f(x) = \frac{x^3}{x^2 - 1} \)
14. \( f(x) = \ln(3 + x) \)
15. \( f(x) = \frac{1}{e^x - 1} \)
16. \( f(x) = \frac{-2}{(x - 2)^2} \)
17. \( f(x) = \frac{x}{x^2 + x - 2} \)
18. \( f(x) = \frac{1}{(x + 3)^4} \)
19. \( f(x) = \tan 2x \)
20. \( f(x) = \sec \pi x \)
21. \( f(x) = \frac{x^3 + 1}{x + 1} \)
22. \( f(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2} \)
23. \( f(x) = \frac{x}{\sin x} \)
24. \( f(x) = \frac{\tan x}{x} \)

In Exercises 25–28, determine whether the function has a vertical asymptote or a removable discontinuity at \( x = -1 \).

25. \( f(x) = \frac{x^2 - 1}{x + 1} \)
26. \( f(x) = \frac{x^2 - 6x - 7}{x + 1} \)

27. \( f(x) = \frac{x^2 + 1}{x + 1} \)
28. \( f(x) = \frac{\sin(x + 1)}{x + 1} \)

In Exercises 29–42, find the limit.

29. \( \lim_{x \to -2} \frac{x - 3}{x^2 - 2} \)
30. \( \lim_{x \to 1} \frac{2 + x}{1 - x} \)
31. \( \lim_{x \to 1} \frac{x^2}{x^2 - 16} \)
32. \( \lim_{x \to 1/2} \frac{x^2}{4x^2 + 16} \)
33. \( \lim_{x \to 3} \frac{x^2 + 2x - 3}{x^2 + x - 6} \)
34. \( \lim_{x \to -3} \frac{6x^2 + x - 1}{x^2 + 4x - 4x - 3} \)
35. \( \lim_{x \to 0} \left( 1 + \frac{1}{x} \right) \)
36. \( \lim_{x \to 0} \left( x^2 - \frac{1}{x} \right) \)
37. \( \lim_{x \to 0} \frac{2}{\sin x} \)
38. \( \lim_{x \to 0} \frac{-2}{\sin \left( \frac{x}{2} \right)} \)
39. \( \lim_{x \to 0} \frac{x^2 - x}{x^2 + 1(x - 1)} \)
40. \( \lim_{x \to 0} \frac{x - 2}{x^2} \)
41. \( \lim_{x \to -3} \ln |\cos x| \)
42. \( \lim_{x \to 0^+} e^{-0.5x} \sin x \)

In Exercises 43 and 44, use a graphing utility to graph the function and determine the one-sided limit.

43. \( f(x) = \frac{1}{x^2 - 25} \)
44. \( f(x) = \frac{1}{x^2} \)

45. Suppose that \( Q \) varies inversely as the square root of \( t - 4 \). Find the limit of \( Q \) as \( t \to 4^+ \).

46. **Boyle's Law** For a quantity of gas at a constant temperature, the pressure \( P \) is inversely proportional to its volume \( V \). Find the limit of \( P \) as \( V \to 0^+ \).

47. **Moving Ladder** A 25-foot ladder is leaning against a house (see figure). If the base of the ladder is pulled away from the house at a rate of 2 feet per second, the top will move down the wall at a rate of \( r = \frac{2x}{\sqrt{625 - x^2}} \) ft/sec.

Let \( x \) be the distance the base of the ladder is from the house.

a. Find the rate when \( x \) is 7 feet.
b. Find the rate when \( x \) is 15 feet.
c. Find the limit of \( r \) as \( x \to 25^- \).

**Figure for 47**
33. Nonremovable discontinuity at \( x = 0 \)
35. Removable discontinuity at \( x = 1 \); 
   Nonremovable discontinuity at \( x = -2 \)
37. Nonremovable discontinuity at \( x = -2 \)
39. Continuous for all real \( x \)
41. Nonremovable discontinuity at \( x = 2 \)
43. Continuous for all real \( x \)
45. Nonremovable discontinuities at integer multiples of \( \frac{\pi}{2} \)
47. Nonremovable discontinuities at each integer
49. \( a = 2 \)  
51. \( a = 4 \)
53. Continuous for all real \( x \)
55. Nonremovable discontinuities at \( x = 1 \) and \( x = -1 \)
57. Continuous on \(( -\infty, \infty )\)
59. Continuous on \( \ldots, (-2\pi, 0), (0, 2\pi), (2\pi, 4\pi), \ldots \)
61. Nonremovable discontinuity at each integer.

63. Discontinuous at \( x = 3 \)

65.

It is not obvious from the graph that the function is discontinuous at \( x = 0 \).

67.

69. \( f(x) \) is continuous on \([2, 4]\).
   \( f(2) = -1 \) and \( f(4) = 3 \)
   By the Intermediate Value Theorem, \( f(c) = 0 \) for at least one value \( c \) between 2 and 4.

71. 0.68  
73. \( f(3) = 11 \)  
75. \( f(2) = 4 \)
77. Discontinuous at every positive integer

79. Discontinuous at every even positive integer

83. True  
84. True
85. False, the rational function \( f(x) = p(x)/q(x) \) has at most \( n \) discontinuities where \( n \) is the degree of \( q(x) \).
86. False, it is discontinuous at \( x = 1 \).
87. Discontinuous at \( x = \pm 1, \pm 2, \pm 3, \ldots \)
93. a. Domain: \(( -\infty, 0 ), (0, \infty )\)
b. 

c. \( \lim_{x \to 0} f(x) = 4, \quad \lim_{x \to -\infty} f(x) = 0 \)
d. \( 4/x \) approaches \( -\infty \) as \( x \to 0^- \) which implies that \( 2^{4/x} \) approaches 0. \( 4/x \) approaches \( \infty \) as \( x \to 0^+ \) which implies that \( 2^{4/x} \) approaches \( \infty \).

Section 1.6 (page 111)
1. \( \lim_{x \to 2^+} \frac{1}{(x + 2)^2} = \infty \)
3. \( \lim_{x \to -\infty} \tan \frac{\pi x}{4} = -\infty \)
\[ \lim_{x \to -\infty} \frac{1}{(x + 2)^2} = \infty \]
\[ \lim_{x \to \infty} \frac{\pi x}{4} = \infty \]
5. \( \lim_{x \to -3} \frac{1}{x^2 - 9} = \infty \)
7. \( \lim_{x \to -3} \frac{x^2}{x^2 - 9} = \infty \)
\[ \lim_{x \to -3} \frac{1}{x^2 - 9} = \infty \]
\[ \lim_{x \to \infty} \frac{x^2}{x^2 - 9} = \infty \]
9. \( x = 0 \)
11. \( x = 2, x = -1 \)
13. \( x = \pm 1 \)
15. \( x = 0 \)
17. \( x = -2, x = 1 \)
19. \( x = \frac{\pi}{4} + \frac{n\pi}{2}, n \) an integer
21. No vertical asymptote
23. \( x = n\pi, n \) a nonzero integer
25. Removable discontinuity at \( x = -1 \)
27. Vertical asymptote at \( x = -1 \)
29. \( -\infty \)
31. \( \infty \)
33. \( \frac{1}{x} \)
35. \(-\infty \)
37. \(-\infty \)
39. \( -\infty \)
41. \(-\infty \)
43. \(-\infty \)
45. \( \infty \) if the constant of proportionality \( k \) is positive and \( -\infty \) if \( k \) is negative.