Examples of Solving Exponential Equations

**Steps for Solving Exponential Equations with the Same Base**

**Step 1:** Determine if the numbers can be written using the same base. If so, go to Step 2. If not, stop and use Steps for Solving an Exponential Equation with Different Bases.

**Step 2:** Rewrite the problem using the same base.

**Step 3:** Use the properties of exponents to simplify the problem.

**Step 4:** Once the bases are the same, drop the bases and set the exponents equal to each other.

**Step 5:** Finish solving the problem by isolating the variable.

**Solving Exponential Equations with Different Bases**

**Step 1:** Determine if the numbers can be written using the same base. If so, stop and use Steps for Solving an Exponential Equation with the Same Base. If not, go to Step 2.

**Step 2:** Take the common logarithm or natural logarithm of each side.

**Step 3:** Use the properties of logarithms to rewrite the problem. Specifically, use Property 5 which says \( \log_a x^y = y \log_a x \).

**Step 4:** Divide each side by the logarithm.

**Step 5:** Use a calculator to find the decimal approximation of the logarithms.

**Step 6:** Finish solving the problem by isolating the variable.

**Example** – Solve: \( 9^{4x-1} = 27^{5-x} \)

\[
9^{4x-1} = 27^{5-x} \quad \text{Determine if } 9 \text{ and } 27 \text{ can be written using the same base. In this case both } 9 \text{ and } 27 \text{ can be written using the base } 3.
\]

\[
\left(3^2\right)^{4x-1} = \left(3^3\right)^{5-x} \quad \text{Rewrite the problem using the same base.}
\]

\[
3^{8x-2} = 3^{15-3x} \quad \text{Use the properties of exponents to simplify the exponents, when a power is raised to a power, we multiply the powers.}
\]

\[
8x - 2 = 15 - 3x \quad \text{Since the bases are the same, we can drop the bases and set the exponents equal to each other.}
\]

\[
x = \frac{17}{11} \quad \text{Finish solving by adding } 2 \text{ to each side, adding } 3x \text{ to each side, and then dividing each side by 11.}
\]

Therefore, the solution to the problem \( 9^{4x-1} = 27^{5-x} \) is \( x = \frac{17}{11} \).
Example – Solve: $2^{5x-2} = 176$

$2^{5x-2} = 176$

Determine if 2 and 176 can be written using the same base. In this case 2 and 176 cannot be written using the same base, so we must use logarithms.

$log(2^{5x-2}) = log(176)$

Take the common logarithm or natural logarithm of each side.

$(5x - 2)(log 2) = log 176$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$5x - 2 = \frac{log 176}{log 2}$

Divide each side by log 2.

$5x - 2 \approx 7.459432$

Use a calculator to find log 176 divided by log 2. Round the answer as appropriate, these answers will use 6 decimal places.

$x \approx 1.891886$

Finish solving the problem by adding 2 to each side and then dividing each side by 5.

Therefore, the solution to the problem $2^{5x-2} = 176$ is $x \approx 1.891886$.

Example – Solve: $e^{7x+5} = 642$

$e^{7x+5} = 642$

Determine if e and 642 can be written using the same base. In this case e and 642 cannot be written using the same base, so we must use logarithms.

$ln(e^{7x+5}) = ln(642)$

Take the common logarithm or natural logarithm of each side. In this case, we should use the natural logarithm because the base is e.

$(7x + 5)(ln e) = ln 642$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$(7x + 5)(1) = ln 642$

Use the properties of logarithms to find ln e. Property 2 states that ln e = 1.

$7x + 5 \approx 6.464588$

Use a calculator to find ln 642. Round the answer as appropriate, these answers will use 6 decimal places.

$x \approx 0.209227$

Finish solving the problem by subtracting 5 from each side and then dividing each side by 7.

Therefore, the solution to the problem $e^{7x+5} = 642$ is $x \approx 0.209227$. 
Example – Solve: \(4^{2x+3} = \left(\frac{1}{32}\right)^{3x+1}\)

Determine if 4 and 1/32 can be written using the same base. In this case both 4 and 1/32 can be written using the base 2.

\[
\left(2^2\right)^{2x+3} = \left(2^{-5}\right)^{3x+1}
\]

Rewrite the problem using the same base. Note that \(1/32 = 1/2^5 = 2^{-5}\).

\[
\left(2^2\right)^{2x+3} = \left(2^{-5}\right)^{3x+1}
\]

Use the properties of exponents to simplify the exponents, when a power is raised to a power, we multiply the powers.

\[
2^{4x+6} = 2^{-15x-5}
\]

Since the bases are the same, we can drop the bases and set the exponents equal to each other.

\[
x = \frac{-11}{19}
\]

Finish solving by subtracting 6 from each side, adding 15x to each side, and then dividing each side by 19.

Therefore, the solution to the problem \(4^{2x+3} = \left(\frac{1}{32}\right)^{3x+1}\) is \(x = \frac{-11}{19}\).

Example – Solve: \(e^{4-7x} = 871\)

Determine if e and 871 can be written using the same base. In this case e and 871 cannot be written using the same base, so we must use logarithms.

\[
\ln\left(e^{4-7x}\right) = \ln(871)
\]

Take the common logarithm or natural logarithm of each side. In this case, we should use the natural logarithm because the base is e.

\[
(4-7x)\ln(e) = \ln(871)
\]

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

\[
(4-7x)\ln(1) = \ln(871)
\]

Use the properties of logarithms to find \(\ln(e)\). Property 2 states that \(\ln(e) = 1\).

\[
4-7x \approx 6.769642
\]

Use a calculator to find \(\ln(871)\). Round the answer as appropriate, these answers will use 6 decimal places.

\[
x \approx -0.395663
\]

Finish solving the problem by subtracting 4 from each side and then dividing each side by \(-7\).

Therefore, the solution to the problem \(e^{4-7x} = 871\) is \(x \approx -0.395663\).
Example – Solve: $7^{4x+3} = 523$

$7^{4x+3} = 523$

Determine if 7 and 523 can be written using the same base. In this case 7 and 523 cannot be written using the same base, so we must use logarithms.

log($7^{4x+3}$) = log(523)

Take the common logarithm or natural logarithm of each side.

(4x + 3)(log 7) = log 523

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$4x + 3 = \frac{\log 523}{\log 7}$

Divide each side by log 7.

$4x + 3 \approx 3.216789$

Use a calculator to find log 523 divided by log 7. Round the answer as appropriate, these answers will use 6 decimal places.

$x \approx 0.054197$

Finish solving the problem by subtracting 3 from each side and then dividing each side by 4.

Therefore, the solution to the problem $7^{4x+3} = 523$ is $x \approx 0.054197$.

Example – Solve: $3e^{5x-9} = 292$

$e^{5x-9} = \frac{292}{3}$

To solve this problem we need to divide by 3 first, and then determine if e and $292/3$ can be written using the same base. In this case e and $292/3$ cannot be written using the same base, so we must use logarithms.

$\ln\left(e^{5x-9}\right) = \ln\left(\frac{292}{3}\right)$

Take the common logarithm or natural logarithm of each side. In this case, we should use the natural logarithm because the base is e.

$(5x-9)(\ln e) = \ln\left(\frac{292}{3}\right)$

Use the properties of logarithms to rewrite the problem. Move the exponent out front which turns this into a multiplication problem.

$(5x-9)(1) = \ln\left(\frac{292}{3}\right)$

Use the properties of logarithms to find ln e. Property 2 states that $\ln e = 1$.

$5x - 9 \approx 4.578142$

Use a calculator to find ln($292/3$). Round the answer as appropriate, these answers will use 6 decimal places.

$x \approx 2.715628$

Finish solving the problem by adding 9 to each side and then dividing each side by 5.

Therefore, the solution to the problem $3e^{5x-9} = 292$ is $x \approx 2.715628$. 